DOCUMENT RESUME

ED 372 298 CE 066 943

AUTHOR	Leonelli, Esther, Ed.; And Others
TITLE	Implementing the Massachusetts Adult Basic Education Math Standards: Our Research Stories. The ABE Math Standards Project. Volume 2.
INSTITUTION	Holyoke Community Coll., MA.; Massachusetts State Dept. of Education, Quincy.
SPONS AGENCY	National Inst. for Literacy, Washington, DC.
PUB DATE	94
NOTE	264p.; For related documents, see CE 066 942-944.
PUB TYPE	Guides - Non-Classroom Use (055)
EDRS PRICE	MF01/PC11 Plus Postage.
DESCRIPTORS	Adult Basic Education; *Adult Literacy; *Classroom Techniques; Curriculum Development; Educational Research; Literacy Education; *Mathematical Applications; *Mathematics Instruction; Research Projects; State Curriculum Guides; *State Standards; *Teaching Methods
IDENTIFIERS	*Massachusetts; NCTM Curriculum and Evaluation Standards; NCTM Professional Teaching Standards

ABSTRACT

The product of a project conducted in Massachusetts to apply the National Council of Teachers of Mathematics' (NCTM) "Curriculum and Evaluation Standards for School Mathematics" to adult basic education (ABE) learning environments, this volume is a collection of teacher-researchers' essays on field-based application of the adapted standards. The following essays are included: "Can the Fear of Manipulatives Be Overcome?" (Debra Richard); "Making Connections in Math and Science" (Karen DeCoster); "'It's Not through the Book'" (Leonora E. Thomas); "Exploring the Metric System in a Workplace Education Class" (Judith Sulzbach); "Non-Traditional Problems in the Forefront" (Marilyn Moses); "Learning to Learn. How Long Does It Take?" (Leslie Arriola); "Manipulatives vs. Rote Memory" (Margaret Fallon); "When You Have a Problem, Use Your Head (...and Your Instinct)" (Thomson Macdonald); "'They Never Asked Me: Who Did They Ask?'" (Shelley Bourgeois, Martha Merson); "'What If I Used Research and the Scientific Method to Teach Problem Solving?'" (Catherine M. Coleman); "Journey into Journal Jottings" (Donna Curry); "Shapes and Stitches: Quilting in an ABE Math Class" (Linda Huntington); "When Students Work in Teams" (Barbara Blake Goodridge); "Learning by Doing, or What Happens When Students Write Their Own Math Word Problems?" (Tricia Donovan); "Evaluating Creative Mathematical Thinking and the GED [General Educational Development Test]" (Sally Spencer); and "Adult Diploma Program Math Research" (Susan Barnard, Kenneth Tamarkin). (KC)

****
* Reproductions supplied by EDRS are the best that can be made *
* from the original document. *
****

ED 372 298

THE ABE MATH STANDARDS PROJECT

VOLUME 2:

IMPLEMENTING THE MASSACHUSETTS ADULT BASIC EDUCATION MATH STANDARDS: *OUR RESEARCH STORIES*

U.S. DEPARTMENT OF EDUCATION
Office of Educational Research and Improvement
EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)

☑ This document has been reproduced as received from the person or organization originating it.

☐ Minor changes have been made to improve reproduction quality.

- Points of view or opinions stated in this document do not necessarily represent official OERI position or policy.

CE 066 943

BEST COPY AVAILABLE

THE ABE MATH STANDARDS PROJECT

is a field-based research project for teacher development
based on the application of
The National Council of Teachers of Mathematics
CURRICULUM AND EVALUATION STANDARDS
FOR SCHOOL MATHEMATICS
to Adult Basic Education Learning Environments

The main funding came as an award from
THE NATIONAL INSTITUTE
FOR LITERACY GRANTS PROGRAM

to

Holyoke Community College

in collaboration with

SABES
System for Adult Basic Education Support

and

The Massachusetts Department of Education

Copyright c 1994 by The Massachusetts ABE MATH TEAM

Permission to photocopy limited material from ***The ABE Math Standards Project, Volumes 1 and 2****, is granted for educational purposes. Permission must be sought for commercial use of content from this publication, when the material is quoted in advertising, when portions are used in other publications, or when charges for copies are made. Use of material other than those cases described should be brought to the attention of The Massachusetts ABE Math Team, c/o Mary Jane Schmitt, Massachusetts Department of Education, 350 Main Street, Malden, MA 02148-5023.*

VOLUME 2:

IMPLEMENTING THE MASSACHUSETTS ADULT BASIC EDUCATION MATH STANDARDS:

OUR RESEARCH STORIES

Edited by Esther Leonelli, Martha W. Merson, and Mary Jane Schmitt

and members of the Massachusetts ABE Math Team

William Arcand
PROJECT CO-DIRECTOR
SABES WESTERN REGIONAL SUPPORT CENTER,
HOLYOKE COMMUNITY COLLEGE

Leslie Arriola
HOLYOKE ADULT LEARNING OPPORTUNITIES
(HALO) CENTER

Shelley Bourgeois
JACKSON-MANN COMMUNITY CENTER,
ALLSTON

Catherine Coleman
WORCESTER ADULT LEARNING CENTER

Donna Curry
SKILLS DEVELOPMENT PROGRAM,
DIGITAL EQUIPMENT CORPORATION

Karen DeCoster
HOLYOKE ADULT LEARNING OPPORTUNITIES
(HALO) CENTER

Tricia Donovan
FRANKLIN/HAMPSHIRE EMPLOYMENT &
TRAINING CONSORTIUM, GREENFIELD

Margaret Fallon
LAWRENCE ADULT LEARNING CENTER

Barbara Blake Goodridge
LOWELL ADULT EDUCATION

Linda Huntington
COMMUNITY LEARNING CENTER, CAMBRIDGE

Esther D. Leonelli
PRODUCT COORDINATOR
COMMUNITY LEARNING CENTER, CAMBRIDGE

Thomson Macdonald
HAITIAN MULTI-SERVICE CENTER,
DORCHESTER

Martha W. Merson
ADULT LITERACY RESOURCE INSTITUTE,
BOSTON

Adrienne Morris
PROJECT ASSISTANT
SABES WESTERN REGIONAL SUPPORT CENTER,
HOLYOKE COMMUNITY COLLEGE

Marilyn Moses
BROCKTON ADULT LEARNING CENTER

Debra Richard
ADULT BASIC EDUCATION PROGRAM,
QUINSIGAMOND COMMUNITY COLLEGE,
WORCESTER

Mary Jane Schmitt
PROJECT CO-DIRECTOR
MASSACHUSETTS DEPARTMENT OF EDUCATION

Ruth Schwendeman
PRODUCT COORDINATOR
OFFICE OF WORKPLACE EDUCATION,
QUINSIGAMOND COMMUNITY COLLEGE,
WORCESTER

Sally Spencer
THE CARE CENTER, HOLYOKE

Judith Sulzbach
QUINSIGAMOND COMMUNITY COLLEGE,
WORCESTER

Kenneth Tamarkin
SOMERVILLE CENTER FOR ADULT LEARNING
EXPERIENCES (SCALE)

Leonora Thomas
NEW BEDFORD ADULT EDUCATION PROGRAM/
ADULT LEARNING CENTER

ACKNOWLEDGMENTS

The Massachusetts ABE Math Team teachers wish to acknowledge the two hundred adult learners who explored strategies for teaching and learning mathematics during this year-long project.

We thank them for their support and healthy skepticism as we tried to create environments that were unlike the math classes any of us had known in our youth. The names of all learners referred to or quoted in this document have been changed to respect their privacy.

Again, we wish to thank the agencies which financially supported our efforts: the National Institute for Literacy which provided the main funding; the Massachusetts Department of Education's Project PALMS (Partnerships Advancing the Learning of Mathematics and Science) which funded one co-director's time and contributed to five of the team members' stipends; the SABES Regional Support Center at Holyoke Community College which funded the other co-director's time; and the SABES Central Resource Center at World Education which helped us conceptualize the project and write our initial proposal.

Thanks, too, to the agencies and individuals who lent moral support, encouragement, and endorsement at key points: the SABES Regional Support Centers, Susan Lytle, and advisory committee members John Comings, Myrna Manly, William Masalski, Bonnie Mullinix (also an honorary team member), and Sally Waldron.

Finally, we thank Mary Lindquist, the President of the National Council of Teachers of Mathematics, who encouraged us. We are indebted to our partners, spouses, and children for support during long days and late nights.

DESIGN AND PRODUCTION BY RICK SCHWARTZ/COMMUNICATIONS

FOREWORD

By Susan L. Lytle, Graduate School of Education, University of Pennsylvania

Over the past decade there has been growing support for the notion that research by teachers about their own classroom practices can function as a powerful means of professional development and also contribute to knowledge generation in the field of education. The terms "teacher research" or "teacher inquiry" have been used as umbrellas to describe a range of activities in which teachers draw on their intellectual histories and close observations of learners to identify questions critical to teaching in specific contexts. Traceable to the writing of educators early in the century, the current efforts of teacher researchers echo John Dewey's belief in the importance of teacher's reflecting on their practices and integrating their observations into their emerging theories of teaching and learning. In his writing, Dewey urges educators to be both consumers and producers of knowledge about teaching, both teachers and students of classroom life.

As more teachers at all levels have come to regard themselves as researchers in their classrooms, schools and programs, they have also begun to explore innovative forms and formats for documenting their own work an for making visible the ways students and teachers together construct knowledge and curriculum. Writing about classrooms as sites of inquiry has thus begun to make accessible some of the interpretive frames of teachers largely missing to date from the literature on teaching. From their 'emic' or insider perspectives, teachers are now examining what happens when inquiry is both subject and method, i.e., when teachers inquire with their students in ways that generate knowledge useful to themselves and their immediate communities as well as potentially useful to the wider field.

In our recent collaborative work, Marilyn Cochran-Smith and I have posited a working definition of teacher research as "systematic and intentional inquiry carried out by teachers." By "systematic," we refer primarily to ways of gathering and recording information, documenting experiences inside and outside of classrooms, and making some kind of written record. The word 'systematic' also indicates ordered ways of recollecting, rethinking, and analyzing classroom events for which there may be only partial or unwritten records. "Intentional" signals that teacher research is an activity that is planned rather than spontaneous, and the use of the term "inquiry" emphasizes that teacher research both stems from and generates questions and reflects teachers' desires to make sense of their experiences by adopting a learning stance toward classroom life.

Despite the growing interest in the movement for teacher research and an expanding arena for publication, however, there continue to be several local and national obstacles, including the isolation of teachers, their occupational socialization as self-sufficient and independent, and the prevalence of a technical model of professionalization in which teachers are expected to be consumers of outside-expert knowledge. In the field of adult literacy education these issues of teacher culture and the dramatic differences among learners and program structures and settings makes the generation of knowledge by field-based practitioners extremely complex. There is much that works against raising questions and against taking the risks of interrogating assumptions and common practices. Overcoming obstacles such as these requires support for groups of adult literacy teachers to come together as inquiry communities that function as intellectually stimulating and challenging context within which to conduct significant research designed primarily to improve daily practice.

The essays included in this collection by teachers of adults in Massachusetts demonstrate the cumulative power of such collaborative inquiry. Building on a National Institute for Literacy grant and embracing a shared overarching question — what happens when adult education teachers try to implement their NCTM-based Massachusetts Adult Basic Education Math Standards in a range of communities and program contexts? — these educators construct specific questions from the particulars of their settings, providing richly detailed accounts of the origins of these concerns, the processes of teacher planning, and the fascinating nuances of student response. Based on close observations of classrooms and students' processes of learning, these narratives capture the energy of highly dedicated and risk-taking professionals by showing how innovation leads not just to decentering the teacher-led classroom or to more student-student interaction but to the development of an inquiry stance that aims to embrace teachers and learners in a common endeavor. From case studies of individuals and projects, close descriptions of collaborations with learners and other teachers, and analyses of efforts to build meaningful cross-disciplinary linkages, we see the many innovative ways that mathematics teachers in diverse communities in Massachusetts are struggling against the powerful legacy of student histories and expectations to integrate the content of math into learners' real-life situations and needs.

As teacher inquiry, this collection contributes to research on adult literacy education by making the distinctive perspectives and knowledge of committed teachers accessible to the field. Using a wide range of interpretive strategies for data collection, including observations, field notes, interviews, audiotape recording, and think-aloud protocols, these descriptions reveal the profound diversity of adult students' needs and interests, the salience of context to teaching method and outcome, and the critical need for educators to interrogate and find alternatives to conventional modes of curriculum and testing. Through their writing, these teachers also disclose how *they* are learning about the subject of mathematics from looking closely at their own curriculum and interactions with learners.

By connecting this community of teacher researchers with others across the country, this publication strengthens the possibilities for generating useful knowledge from a field-based perspective. It is extremely important that adult educators make visible their work in progress in collections that reinforce the need for local rather than monolithic solutions to the challenges of educating adults through the words of these teachers and learners using the NCTM and their own standards as a heuristic to interrogate and document their daily work, so that others in the field of adult literacy education — including teachers, learners, administrators, university-based researchers and policy-makers — can understand what it means to use a powerful overarching framework to improve practice in ways that are respectful of and responsive to differences among learners, teachers and communities.

INTRODUCTION

By Martha Merson, Adult Literacy Resource Institute

When teachers read the *NCTM Curriculum and Evaluation Standards for School Mathematics* or the Massachusetts Adult Basic Education Math Standards drafted by the Massachusetts ABE Math Team, they might have mixed reactions. While many of us on the Math Team were at once excited by new possibilities when we heard of or read the *NCTM Standards*, we also felt awed or even resigned ahead of time that such a different way of teaching and learning could ever occur in our own classrooms. The teachers who have written for this book have overcome those fears. They've attempted changes in the format, the content, the very spirit of their classes. In this volume we've outlined our journeys. Each article tells the story of a question. These questions go to the very heart of our teaching. We ask ourselves to question our "best practice" — to risk disrupting our own status quos in order to create mathematical communities where we problem-solve, reason, seek connections, and communicate with one another.

Be aware that the changes discussed here were not implemented in ideal settings by privileged teachers. In almost every article, teachers remind us of the constraints common to most adult literacy and basic education: shaky attendance patterns, lack of prep time, open entry and open exit enrollment, and multi-level classes. This was the norm. Nevertheless, teachers who had a choice among classes, and those who didn't, worked with some of what one might in advance have labeled the hardest cases: the strictly individualized GED class became teams, the beginning level math and reading students learned geometry and statistics, and students who were loyal to the culture of schooling that had trained them to compute answers began to write questions and demonstrate reasoning.

When teachers began initiating changes in their classes, they did so with (some) guidance from me designed to facilitate their growth into teacher researchers.

Teacher research suited the projects the Math Team teachers took on in three ways. First, an underlying assumption of teacher research is that no teacher can change his or her classroom singlehandedly. Change in classrooms is a process shaped as much by learners and their contexts as it is by teachers.

Secondly, teacher research allows for and encourages learning on the part of the teachers. The point of the project was not rubberstamp practices to which we already subscribed. Our aim was to apply the Standards in the reality of teaching in various adult education environments.

Thirdly, perhaps most importantly, as teacher researchers, the projects were an opportunity to experiment. No one could fail —***if*** something was learned. Many on the ABE Math Team took the opportunity to face a longstanding challenge. Lytle notes that undertaking teacher research involves thinking and writing and making change in ways that are not inherently safe. She writes: "...Many teachers are making their research risky business. They are taking on the hard social and political issues that inform instruction, surfacing deep questions that involve disclosure of the intimacies of their daily practice..." (See Lytle, "Risky Business," *The Quarterly*, Vol. 15, No. 1, Winter, 1993, pp.20-23). Mary Jane Schmitt described us as finding the cliff and forcing ourselves to jump. Of course she believed we'd fly or at least find the drop only slightly unsettling.

For those who would like to undertake similar study of their classrooms or the *Standards*, this offers a brief description of the steps we took, knowing that this is only one perspective. For a more complete description of the entire project, including the process for studying the NCTM *Standards* and developing the *Massachusetts Adult Basic Education Math Standards*, see Chapter 1 of that volume. While every teacher involved had his or her own question and carried it out in his or her own setting, much of the conceptualizing of the inquiry questions, the decisions about data collection and the analysis of the data were interactive processes that occurred either in the small work groups or the large group meetings or both.

Teacher research has been defined by Cochran-Smith and Lytle as "systematic, intentional inquiry" *(see Lytle and Cochran-Smith "Teacher Research as a Way of Knowing". Harvard Educational Review, Vol. 62, No. 4, Winter 1992, p. 450)*. At every stage of the process, it has been imperative for us to hold onto a common vision of teacher research. It is particularly important for us to put forth the demands of the genre as we have understood and constructed them, and to demonstrate the extent to which the papers conform to this definition. For it is not our intent to reach an audience of academic education researchers with "research findings" that would be acceptable to highly trained quantitative or qualitative researchers. We aim instead to reach teachers like ourselves who have questions about our practice that can best be answered by us: insiders in the classroom community. Indeed, we did remain true to the standards set for this research genre in theory by Cochran-Smith and Lytle in "Teacher Research as a Way of Knowing" and in practice in Lytle, Belzer, and Reumann's facilitation of the Adult Literacy Practitioner Inquiry Project based in Philadelphia in 1991-1992.

Key to "systematic, intentional inquiry" is the "inquiry question." Each Math Team member formed a question that combined an interest in a mathematical topic area, a standard, and that targeted a particular environment or student population. These questions, stated in the beginning of the papers, range from long-term and open-ended "How can I encourage curiosity?" or "Will teaching about statistics encourage students to question?" to almost rhetorical: "Can the fear of manipulatives be overcome?" As in much qualitative research, these questions served as a beginning point from which teachers could travel, depending on class interactions, changing dynamics and interests. The first section of most papers describes not only how the questions were reached, but in cases like Peg Fallon's and Susan Barnard and Kenny Tamarkin's, the history of a question that has been a nagging concern or in Marilyn

Moses', the spirit of experimentation, or in Lee Thomas', the revolutionary thinking behind a certain approach.

Team members made plans to collect information in their classrooms that could answer the inquiry question. Teachers opted to collect data by one or more means: taping, keeping a teaching journal, asking students to keep learning logs, interviews, and think-aloud protocols. Most found themselves rewarded by the additional effort required to coordinate yet another aspect of classroom life. In Linda Huntington's article, she wishes she had taped more often. Sally Spencer acknowledges that she learned most from think-aloud protocols, and Leslie Arriola found her co-teacher's notes on class invaluable. In Judy Sulzbach's case, data collection extended beyond the classroom to the workplace. She interviewed supervisors (about the math on the worksite). While "systematic" is inherent in the definition to which we subscribe, some teachers were 100% systematic. Donna Curry, Tricia Donovan, and Barbara Goodridge religiously taped or had their learners write. Other teachers, like Cathy Coleman, Karen DeCoster, and Linda Huntington, found that in spite of their best efforts, the data they had on their students varied according to attendance. Exciting conversations happened when the tape recorder wasn't on. If data collection wasn't perfectly systematic, the teacher research process provided another opportunity for us to be systematic.

The descriptive review processes developed by Pat Carrini and the teachers at the Prospect School in Vermont are a systematic way to analyze data. The processes demand structured responses to and allow for a group of listeners to raise creative questions about the data. Using these processes, the group interrogated words like "problems", raised questions about small group interactions, and used each other to confirm insights and instincts. The revision stage of the writing process provided team members with another opportunity to think about the most important findings and the implications for their own practice and for the field.

The evidence demonstrating how the purposes of teacher research were met in this project is visible throughout this manuscript. To summarize here: Teachers addressed questions that were essential to them rather than framed by a research agenda put forth by others; a powerful staff development model took root as teachers laid aside their investment in proving they could teach according to the Standards and began to take risks; and the voices of those closest to adult learners have discovered a voice and a place to record the truths they have found in their classrooms; and finally, it is hoped, have found a valued audience in you: teachers, administrators, policy-makers, funders and others.

TABLE OF CONTENTS

Acknowledgments

Foreword

Introduction

MAKING MATHEMATICAL CONNECTIONS 1

Can the Fear of Manipulatives Be Overcome? 2
Debra Richard, Quinsigamond Community College

Making Connections in Math and Science 14
Karen DeCoster, Holyoke Adult Learning Opportunities (HALO) Center

It's Not Through the Book 29
Leonora E. Thomas, New Bedford Adult Education Program

Exploring the Metric System in a Workplace Education Class 48
Judith Sulzbach, Quinsigamond Community College, Worcester

MATHEMATICS AS PROBLEM SOLVING 59

Non-Traditional Problems in the Forefront 60
Marilyn Moses, Brockton Adult Learning Center

Learning to Learn. How Long Does It Take? 78
Leslie Arriola, Holyoke Adult Learning Opportunities (HALO) Center

MATHEMATICS AS REASONING 93

Manipulatives vs. Rote Memory 94
Margaret Fallon, Adult Learning Center of Lawrence

When You Have A Problem, Use Your Head 107
Thomson Macdonald, Haitian Multi-Service Center

They Never Asked Me. Who Did They Ask? 119
Shelley Bourgeois, Jackson-Mann Community Center,
and Martha Merson, Adult Literacy Resource Institute, Boston

What If I Used Research and the Scientific Method to Teach Problem Solving? 133
Catherine M. Coleman, Worcester Adult Learning Center

MATHEMATICS AS COMMUNICATION 153

Journey Into Journal Jottings 154
Donna Curry, Digital Equipment Corporation

Shapes and Stitches: Quilting in an ABE Math Class 176
Linda Huntington, Community Learning Center, Cambridge

When Students Work in Teams 192
Barbara Blake Goodridge, Lowell Adult Education

Learning by Doing, or What Happens When Studens Write Their Own Math Word Problems? 207
Tricia Donovan, Franklin/Hampshire Employment and Training Consortium

ASSESSMENT 221

Evaluating Creative Mathematical Thinking and the GED 222
Sally Spencer, The Care Center, Holyoke

Adult Diploma Program Math Research 242
Susan Barnard and Kenneth Tamarkin,
Somerville Center for Adult Learning Experience (SCALE)

About the Massachusetts ABE Math Team

Afterword

MAKING MATHEMATICAL CONNECTIONS

1

CAN THE FEAR OF MANIPULATIVES BE OVERCOME?

by Debra Richard, Quinsigamond Community College

When educators talk about manipulatives, I picture in my mind brightly colored blocks in primary colors, big and chunky for little hands. These are manipulatives; I am referring to pattern blocks. But manipulatives can be anything that a student can touch and explore, from a piece of paper to a ruler to a computer. Manipulatives can be teacher or student made. They don't have to be something purchased.

> ***"Making (such) connections aids learners as they move from concrete thinking and problem solving to abstract concepts. It also broadens their perspectives and promotes the vision of mathematics as an integrated whole."***
>
> MASSACHUSETTS ADULT BASIC EDUCATION MATH STANDARDS, STANDARD 4: MATHEMATICAL CONNECTIONS

I was drawn to this question because so many educators are now talking about manipulatives and hands-on activities. The theory is that if the student touches, feels, gets to play with, and uses more than just sight (book) and sound (lecture), then the student will learn and thus retain the material better. At first I was skeptical. My concerns were:

"Why bother?"

"Okay, so what if this works for kindergartners; will this work with my adult education classes?"

"Will they be interested?"

"Will they like this?"

"Do I have enough time to let them play with these manipulatives if their goal is to get their GED as soon as possible?"

I wasn't sure.

There are many aspects to my question, "Can the fear of manipulatives be overcome?" The first being, of course, whose fear am I wondering about? It is my belief that both the students and their teachers have a fear of manipulatives if they have never used them. When I asked my students to work with pattern blocks, pieces of paper, or even graph paper, they said to me:

"I don't want to do this."

"I don't have time!"

"Can't we move along a little faster?"

But, after talking with them a bit more, the real concern seemed to be with the unknown.

"Will I be able to do this?"

"Will I be embarrassed?"

During this school year, 1992-1993, I was fortunate to be part of the PALMS (Partnerships Advancing Learning in Math and Science) project and the Massachusetts ABE Math Team. The philosophy of PALMS is that students need to explore science and mathematics concepts instead of just being lectured to. So, again, I was going to be involved with manipulatives. PALMS has made the assumption that learning this way is more beneficial to the student. Again, I wasn't sure.

In all honesty, my intention was to prove that using hands-on material was like the "new math": it would surely be gone tomorrow. Certain that I had the right idea all these years that lecture and note taking was the way to teach, I was determined to show it using my ABE and GED math classes.

CONTEXT

The Adult Basic Education Program that I work in is through Quinsigamond Community College in Worcester, Massachusetts. We work at a neighborhood site in a church's CCD classrooms. We meet three mornings a week for three hours a day. Fifty percent of the time is for math, so students receive math education about 1-1/2 hours, three times a week. Our classrooms are small, and the space is a bit crowded. We do not have tables, which really is a help when working cooperatively with manipulatives. We have desks, which makes pairing up difficult, and moving around these "pairs" a challenge.

I worked with two classes of learners. The first is the GED class, which is made up of students who are predominantly American born. The second class is our middle level class which is made up of a wider variety of nationalities, American-born as well as many from other countries. Most understand English, but many have trouble understanding mathematical language. Their ages range from 16 to 50.

The GED class as a group has a bit more motivation than the second class, but both of the classes were open and excited to be part of the PALMS project. Some had children involved in the project in the elementary grades.

There is an attendance problem in both classes. Many of the adults have other commitments, such as sick children, welfare house inspections, and so on, which certainly has an impact on their motivation.

What they all have in common is a desire to better their own lives, and those with children, to better their children's lives. They are willing to try to learn, even though there are some who admit to hating math.

METHOD AND IMPACT

To answer my question, I thought back about my teaching over the past 12 years in every area of mathematics from junior high school to college. I asked myself, "What did I do that worked?" Lecture was my main form of communication, as it is in many classrooms. Very rarely did I use small groups, or even simple manipulatives. I remembered using a made-up checkbook activity with my seventh graders that was very successful. Also, I remembered asking my GED students to memorize the measurement equivalencies which was never successful. What was the difference? I still wasn't sure.

I learned the most math not as a student, sitting at a desk and taking notes; I learned the most math when I was the teacher, explaining, drawing, talking, and communicating.

Then, I thought about when I learned the most math myself. And then I was sure. I learned the most math not as a student, sitting at a desk and taking notes; I learned the most math when I was the teacher, explaining, drawing, talking, and communicating. That was the difference. I was convinced at that point that my adult students needed to get more involved in their education.

To see how the students were reacting to using manipulatives, I decided to have them write about their experiences and their feelings after each try. On two occasions, I taped the class.

Not knowing exactly how I should begin, but since I was at the point of attempting to teach measurement equivalencies again, I decided one day to set up centers the way that my daughter's first grade teacher does. Will my adult students accept this?

MEASURING

After collecting gallon jugs, yardsticks, a clock, a calendar, a cup, a pint, and a quart, off I went to begin. I set up three centers: weight and length, time, and volume. Students were separated into three groups. We all agreed that the group had to move along as a whole and could not move to the next center unless everyone in that group understood what was to be explored. The students agreed to try to teach each other. See the attached "Teacher Activities" for a more detailed explanation.

Students took water and found that eight ounces were a cup. They poured water into a pint and found that one pint was two cups. They looked at clocks, calendars, and rulers. They explored. It was a little messy, and it took more time than just saying, 'here it is folks, memorize it.' But did it work?

The students told me after class that for the first time they could see what a pint was, that they understood measuring and could picture the equivalencies in their heads. They felt that this idea was good and wanted to do more of this learning.

One of my students, Roberta, came in the next day and told me that she went home and cried about something that she had learned the day before. I felt pretty awful, and asked her what she meant. She said, "I never realized that when I gave my son eight ounces of formula that he was drinking a whole cup of formula." Aha! She made a connection that my students in the past had not.

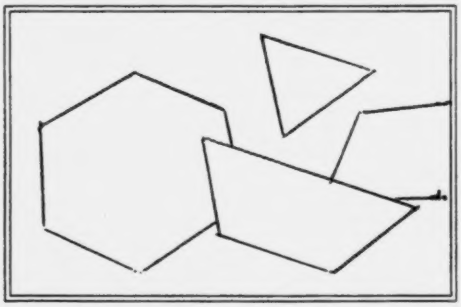
It was powerful for me that many students said that they never quite understood this before. One 16 year old boy, who has lived in foster care for most of his life, said that he had never touched a measuring cup until now! My resistance to manipulatives was going away. I began to feel a slight confidence in approaching the next idea.

One 16 year old boy, who has lived in foster care for most of his life, said that he had never touched a measuring cup until now!

FRACTIONS

We used pattern blocks in pairs to come up with equivalent fractions. After explaining that the yellow hexagon represented a whole, the groups had to come up with what the other pieces represented. Aha! Some students were very unsure what some fractional forms were. All knew 1/2. Most were clear on 1/4. 1/8 was difficult; 1/3 was even harder.

In a follow up lesson, the students, some of who were absent for the pattern block lesson, were asked to draw what "1/2 + 1/4" looked like. My expectation was that the learner may draw a circle and shade 1/2, draw a circle and shade 1/4, and finally draw a circle and shade 3/4 of it. Many of the students could not do this. One wrote:



"This takes too long and is confusing. I don't need to draw it. I only need to do it!"

Another asked me, "Why make us do this?"

It became clear that many students had a huge hole in their math knowledge. Somewhere, and it appears with some right at the beginning, they never received the very basics of math. How powerful this was for me as a teacher to realize that some of my students could not add fractions because they truly had no idea what they were doing.

THE FEAR IS GONE

After this day, my fear of manipulatives was completely gone. Actually I felt very foolish not to have made the connection that the adult learner may have never received such basic instruction as to know what a fractional piece looks like. I felt very foolish to have always approached my adult students with "simple" rules and tricks to "help" with their math instead of realizing that my students had a great deal of difficulty picturing what was happening on paper in their minds. Concepts that seemed so obvious to me were not understood at all. To explain how powerful a revelation this is on paper is impossible. This experience needs to be experienced.

My students wrote on their feelings about manipulatives:

"It seemed hard at first, but once you start to get into it. It is very easy to understand."

"At first it was hard for me. It was a good thing working together. Every day you learn some new!"

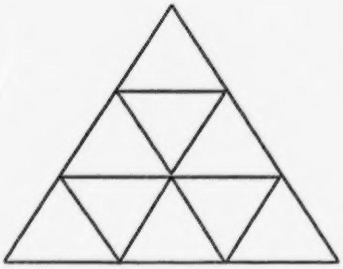
It seems to me that if the ABE learner likes something, they will come back. If they come back to class tomorrow, then we have a chance. If the class is boring to them, if they don't like it, or if they feel uncomfortable, they will not come back, and we lose them. I have found that with the adult learner getting and keeping them is somewhat of a challenge.

I taped the class on two occasions. I thought that these tapings would be embarrassing to the student; I'm uncomfortable when meetings are taped and am a bit less likely to speak up. But the students almost acted up more for these tapings. "Next time put it closer to me!" a usually quiet student asked. Interesting? My students were getting involved.

When I listened to these tapes, I was so surprised to hear the enthusiasm in their voices. They were talking, really staying right on task, and being very concerned with what they were doing.

OUTSIDE THE CLASSROOM

To convince myself that hands-on cooperative teaching is effective, I looked in places other than my classroom. The director of our ABE program, Betsy Zuegg, and I manned a table at Worcester's "Healthy Family Day." She and I set up three hands on activities: making polygons using marshmallows and toothpicks, watching a flower absorb food coloring through its stem which results in the petals changing colors, and brain teasers. We let the people, who were mostly low income, take home what they made and gave a pencil to all who had a correct answer to the brain teasers. Our table was mobbed. The children and their parents enjoyed the activities. One tough-looking 12-year-old wandered over



with a chip on his shoulder. When asked to guess the number of triangles in this picture, he replied, "That's easy. There's nine."

"Look again," we coaxed.

"Well, maybe ten."

"Still more."

His attitude was changing. He wasn't so tough anymore. He was interested.

"There can't be!"

We helped him to find the other three. That boy walked away with a pencil and the biggest smile. He felt really proud of what he had done.

Looking around at the other booths, I noticed that those with something to do were crowded. The tables that had information to pass out were deserted. Think about it. People really like to "do."

At a family outing to Boston's aquarium, I couldn't help but notice that the room where visitors could pick up and touch the starfish and hermit crabs was packed four deep with people waiting. Parents and children wanted to know what these felt like. The exhibits where there was a fish tank to look at was virtually vacant of visitors.

I really did intend to show that manipulatives were a waste of time. How surprised I am to be writing this now, trying to show the reader the importance of manipulatives to the student. But, I feel so fortunate that I had the opportunity to experience what I have this year. If it wasn't for this project, I still would be lecturing and watching my students yawn.

FINDINGS

✔ Teachers need to overcome their resistance to manipulatives first. Once this occurs, the teachers' enthusiasm will be transferred to the learner. Both will be willing to explore.

✔ When my students were lectured to, they would take notes, ask questions, try a few examples, and leave. Very few smiles could be seen. When my students were left alone to explore with hands on material, they took notes, asked questions, and laughed! Students seemed to connect more with their classmates, and some made new friends.

✔ Students made connections between math and their own lives. A fiftyish male machinist could not reduce a fraction or add them or do anything with them. But when he made the connection between fractions and parts of an inch, as on his machinist's ruler, he understood. "I would never say 16/32. I would know to say 1/2!" What a waste of time it would have been to have him suffer through worksheet after worksheet trying to get him to understand. How simple it was to show him a ruler and let him make the connections.

✔ The hardest part for me now when using manipulatives and allowing students to discover is to leave them alone. I have the tendency to try to help them along, which can take something away from the experience. Pulling back and watching and trying not to answer questions is difficult, but students seem to benefit greatly from the experience.

Pulling back and watching and trying not to answer questions is difficult, but students seem to benefit greatly from the experience.

✔ The last "finding" has some ties to teaching, but more so with my life. Instead of being the controller, I now see the benefits to allowing more creativity. This has affected my own children. Instead of trying to design games for them or activities to take them to, I have tried pulling back and letting them create activities for themselves. My three-year-old son now begs his six-year-old sister to play house, even though he does insist on being Batman while doing it.

CONCLUSION

Can your fear of manipulatives be overcome? Yes! Is it worth it? Yes! Do you have the time, even though your GED students have so little time? You cannot do without it.

This paper took you through my journey to try to overcome my resistance to manipulatives. I hope that you are convinced that it is worth the effort or at least open at this point to try.

The gaps in the adult learner's math background need to be addressed. How could anyone picture in their minds something that they have never seen?

Manipulatives increased the number of connections the student made to tools, routine tasks, and basically, to their lives. They began to see that mathematics is in their lives.

During this experience, many other questions surfaced, and I began to try to think of better ways to do lessons. For instance, instead of announcing, "Let's try these manipulatives today," it may be wiser to begin, "Let's explore the odds with these dice." Instead of introducing manipulatives with something unfamiliar, like pattern blocks, begin with something familiar. If you're teaching measuring, ask students to bring in their own measuring tools and go from there.

This is my research, findings, results, and opinion. Although I do not expect this paper to change math instruction for adults entirely, I do hope that this project will be part of the movement to change the way that math is taught. I sincerely hope that no longer will the tools for teaching math be considered a book, a pencil, a paper, and an assignment to do the next 20 problems and check your answers in the back. After doing this project, I now realize why so many people do not like math. Who would like trying to rearrange numbers with absolutely no concept as to what they are doing?

At our "graduation" ceremony at the end of this school year, I called each student from my classes up to the front of the parish hall, one by one, to receive a carnation and a certificate of achievement. When I called

Roberta, whom I spoke of earlier, and announced to everyone in our program that she got her GED, she walked up slowly and gave me a hug. She backed away saying that she wanted to tell everyone a story. How surprised I was to hear what she had to say.

"My first day in Debra's math class, we tried to find how many ounces were in a cup. I went home and told my husband how stupid I was and cried all night. John (her husband) set up bottles and measuring cups, and we poured water all night until I understood... I never liked math before, but I know that I can do it now."

TEACHER ACTIVITIES

1. **TRY THE MEASUREMENT EQUIVALENCY ACTIVITY AS DESCRIBED IN MY PROJECT.** Distribute a prepared sheet where all measurements are listed but do not list the equivalency. You can use the GED equivalency chart or let the students make up this sheet for themselves. For example:

   1 cup = ____ ounces
   1 pint = ____ cups

   Set up the centers, arrange your class into three groups, and let them figure out the equivalencies.

2. **A WONDERFUL ACTIVITY IS THE "JIGSAW PUZZLE," SOURCE UNKNOWN.** Arrange the class into groups with three people in each group. The number of groups that you may have doesn't matter. Each member of the group is assigned a 1, 2, or 3.

   The next step is to have all of the 1's go together, all of the 2's go together, and all of the 3's go together. When they are assembled this way, each of these three groups will learn something that pertains to one topic, but each group learns a different concept. After an allotted time, everyone returns to their group.

   Everyone now tries to teach the other two members of their group what they have learned. The 1's teach what they discovered, then the 2's, and then the 3's.

   I have tried this in Geometry. The 1's explored how many degrees are in a triangle, the 2's learned about ways to classify triangles by sides, and the 3's learned about ways to classify triangles by angles.

   This activity can be done with any topic. The students are involved and pleased when others understand what they have taught.

3. **AREA AND PERIMETER.** Give the students a sheet with different size squares (then try rectangles and triangles) drawn on graph paper. Have the students make a chart that would give the side, area, and perimeter. Let them measure and count the area inside of each figure and record it.

   After some time, ask the students, "If you know the side, then to get the area.............................. and to get the perimeter................................"

   Once the student can make his/her own formula, it will be understood.

4. **CIRCUMFERENCE.** Many adults do not understand *pi*. When using the formula to find the circumference of a circle, some will get lost. Try this:

   Have students work in pairs. Ask each student to bring in a can with a label on it. Slit the label. With a ruler, measure the length of the label and record it. Measure the diameter (going across) of the can. Ask each group to write down what would you multiply the diameter by to make it equal to the length of the label? Most will come up with 3. Since pi=3.14..., the explanation will be a bit easier to understand.

5. **LASTLY, DO NOT OVERLOOK THE RULER AS AN IMPORTANT TEACHING TOOL.** But in order for it to be effective, the student must know how to measure. Spend time showing your students how to read a ruler. Explain to them how the inch is broken up. Explain what all the little lines mean. I was surprised as to how few of my students could read a ruler! Have the students measure designated objects, and then let them compare with each other. Understanding how to read a ruler will be an important life skill for them to possess.

MAKING CONNECTIONS IN MATH AND SCIENCE

TRAVELING TO THE EMERALD CITY

By Karen DeCoster, Holyoke Adult Learning Opportunities (HALO) Center

> ***"In the adult basic education classroom, curriculum design must include approaches to making mathematical connections which allow the learner to apply mathematical thinking and modeling to solve problems that arise in other disciplines, and in the real world, including work-related settings."***
>
> MASSACHUSETTS ADULT BASIC EDUCATION MATH STANDARDS, STANDARD 4: MATHEMATICAL CONNECTIONS

What happens to students' definitions of math and science as they explore the connections between them through an integrated unit approach?"

The *NCTM Curriculum and Evaluation Standards for School Mathematics* and the *Massachusetts ABE Math Standards* call for an emphasis on the connections between mathematics and other disciplines. Connections, it is suggested, should be frequent enough to influence students' appreciation for the value of mathematics in their other studies as well as in the world around them. Additionally, teachers are encouraged to take advantage of students' interests and experiences to foster this appreciation of math. The last part was a cinch; it's the way I've always taught. I can't imagine teaching without students' interests and experiences in mind. But emphasize the connections between math and other disciplines? How many were there?

Although I now consider myself to be a mathematical thinker I really wasn't one when this project began. Math and I had never been close friends. In high school I'd only taken Algebra I and II and geometry. In college it was even more pathetic—I simply took the math required of an elementary ed major—Math in the Elementary Classroom. I taught math as I viewed math, as an essential yet lifeless skill. I turned a deaf ear towards math because it seemed to lack the fascination I had found in several other disciplines. Science was one.

In high school I took chemistry, anatomy, etc. In college it was Biology I and II, anatomy and physiology, and two years of marine biology. I loved science. Consequently, when I was approached to participate in the PALMS Project (Partnership Advancing the Learning of Math and Science) during the 1992 school year I became excited at the prospect of bringing more hands on science into my classroom. Since most of my science teaching came from ABE materials coupled with some low budget creativity on my part, I saw this as "manna from heaven." I guess you could say a very mathematical thing happened...I was interested in half of the PALMS project and hoped the other half would go away.

However, to my benefit, it wasn't going to work that way. As part of my involvement in the PALMS project I was required to join the Massachusetts ABE Math Team and, as part of my involvement in the Math Team, I was asked to do math research. Fine I thought, my question will be one in which science can play a leading role. Little did I know that as my research progressed my relationship with math would change drastically. I soon discovered why the integration of math with contents was an overarching goal of the *NCTM Standards;* it really does "illustrate math's usefulness...and establishes a belief in the utility and value of mathematics." For me it brought it to life!

As I began to confront the question of how an intensive integration of math and science would affect students' definitions of the subjects, a secondary question arose. By teaching two subjects at once could I save time in a teaching environment where five subjects are squeezed into 1.75 hours, four times a week? The answer was easily found but worthy of mention...NO. The hands-on work of knowledge production required that I devote more hours to preparation and that my students spend more time immersed in real learning. Time lost some of its importance for all of us.

THE STUDENTS

I was very fortunate to have an outstanding group of adults as research assistants during the project. They were ten brave learners, five of whom began with me in September and finished with me in June. (Carol-37, Nancy-49, Kyle-23, Bob-24, Kara-21, Tammy-30, Donna-30, Edward-22, Ana-26, Mario-20) While the group was very stable, I did lose Edward in April for unknown reasons and Mario (a student of mine for two years) in early May after he passed the GED (total score 225: math 46, science 44). A new student, Jane, joined

HALO (the adult learning center I teach at in Holyoke) in late March as an independent student and filled Edward's seat in April.

The class was a 50/50 split when it came to math interest. Kara, Edward, Donna, Carol, and Mario preferred math to all other subjects and would do nothing else if left alone. The others felt uncomfortable with math for very different reasons: Bob had been diagnosed as dyslexic; Tammy, having been raised on a remote Philippine island, had little formal math training; Kyle had admittedly "burnt himself out" on drugs and alcohol and found all the "steps and rules impossible to keep in his head"; Ana had "always been lousy at math" (according to her); and Nancy found school, and therefore math, overwhelming in general. Linda, an excellent reader and writer, felt extremely insecure about math and saw it as a GED obstacle.

All 11 students could do basic whole number computation and problem solving successfully, with the exception of Bob who was still struggling through long division. My five "math aficionados" were capable of basic fraction computation but became confused with fraction word problems. The remaining six felt that fractions were very confusing. The entire group did have a basic sense of decimals and percents, some more than others.

At the beginning of the school year, as I experimented with the changes suggested in the *NCTM Standards* (my research had not officially begun) I was met by a rather distrustful group. They would often roll their eyes if I suggested that they do something as tame as use a diagram to solve a given math problem. They resented the fact that I was asking them to depart from the conventional number juggling they knew so well. The arguments went like this:

Donna: "Why should I change the WAY I do math when it works so well for me doing it my way."

Edward: "This is stupid. Kids do it this way."

***Nancy:* "Look, I find math hard enough and now you're asking me to do things in a way I was never taught."**

***Kara:* "It takes longer to think of what I should draw and then draw it. My way is faster."**

Those moments were unsettling for me since I shared some of those feelings. I worried, too, that since I was new to the spirit of the *Standards* perhaps I had approached the lesson incorrectly or chosen the wrong problem. I'm sure on some occasions I had, but we'd sit down and talk about the fact that this was not only a learning process for them but for me as well.

I think watching me make mistakes was one of the most powerful things that happened to the group. As Carol once said in class, "Watching you screw up, YOU THE TEACHER, makes me feel better about when I do it." During discussions of this sort (we had many—some taped) I would read something to them from some mathematically "evangelical" article I had read, and follow it with some personal anecdote about my ongoing math conversion. I wanted them to realize that I needed THEIR help to become a better math teacher. They'd joke about the "big job" they were being asked to do and, as was common in this class, laughter filled the room. (They were so good humored.) What made this group even more remarkable was the way in which they embraced their guinea pig roles in a matter of weeks and settled in for the long journey ahead.

I think watching me make mistakes was one of the most powerful things that happened to the group. As Carol once said in class, "Watching you screw up, YOU THE TEACHER, makes me feel better about when I do it."

WHAT HAPPENED

One of the earliest "*Standards*-flavored" articles influencing my project was "The Futility of Trying to Teach Everything of Importance" by Grant Wiggins. (I picked it up at PALMS meeting.) Reading it for the first time caused the academic ground on which I'd stood to quake. I soon began to question the dominant methodologies I had come to know—especially those that pressured my students and me into stockpiling the "necessary" information for the GED:

> ***"Curriculum design could be liberated from the sham of typical scope and sequence whereby it is assumed that a logical outline of all adult knowledge is translatable into complete lessons...The test of modern curriculum is to enable students to see how knowledge grows out of, resolves, and produces questions. In short, the aim is to awaken, not "stock" or "train" the mind."***

For a brief moment I questioned whether this applied to such goal-oriented students as those seeking to pass the GED. Based on earlier discussions with my GED group I decided that it did. Their goal, they told me, was "to become educated more than it was to pass some test." This conviction framed the integrated math-science curriculum embedded in my research and influenced by Grant Wiggins. Together, the students and I would research the answer to this question: What happens to OUR definitions of math and science when WE explore the connections between them? After all, wasn't I discovering right alongside them?

As assessment tools, we agreed to tape research discussions three times during the project (February, late March, and mid-May) and write our answers to the following questions each time:

What is math?

What is science?

What connections, if any, do you see between the two?

Unfortunately, due to attendance problems and an aversion for make-up work (even in the name of research) I wasn't able to get THREE separate responses from EACH student during the project. I did, however, for 50% of the group.

We also agreed to keep journals in which we would respond to one or both of the following questions:

What did you learn today?

Where was the math in the science and what was the math we used?

In February we began our research project with an "interest inventory" and our first research discussion. To dispel any fears that this project would be a complete departure from the GED I asked students, in pairs, to scan numerous GED science books and form lists of topics that might appear on the test. All the groups' lists were then put on the board into one master list. Once they were sufficiently overwhelmed, I suggested that we narrow things down by selecting those topics that we'd be most interested in exploring together. Using the classic scale of 1-10 (10 indicating very high interest) the students and I assigned each topic a number. Our responses were then recorded alongside each topic, averaged, graphed and later displayed in the room.

We had decided to probe the following science themes from February to May: electricity, chemistry, and astronomy .

At the end of our first research discussion that day the students and I wrote our answers to the questions— What is math? What is science? The following is a representative sampling:

Donna: Math is multiplying, dividing, adding and subtracting and much more. It's also decimals, fractions, percents, and stuff like algebra and geometry, which I can't do.

Bob: Math is arithmetic. You need it for banking and paying bills and carpentry etc. Many jobs require you to be good in math.

Nancy: Math is a bunch of numbers, letters, angles and symbols. I guess you could go on and on! It's very confusing at times because there is so much to learn. It's something I'm not good at...

Kyle: Science is really interesting. It's understanding our world through microscopes and telescopes and sciences like chemistry and biology.

Mario: Science is chemistry, geology, microbiology, astronomy and physiology, psychology and a

lot more that I can't think of right now. Oh yeah, it's also boring.

Karen (teacher): Math is numbers both rational/irrational, operations, strategies, and processes. Math is a survival tool. Some people are better at it than others.

(Edward and Kara were so insecure about writing that they did not participate in this part of the project.)

The connections question was answered collectively and recorded on large sheets to remain displayed in the room. I did not participate in answering this question since I wanted an assessment tool free of my influence. The connections they saw were as follows:

fractions, decimals, and percents — weights and amounts of chemicals are measured with these measurement/weight — ounces and pounds of chemicals and body fat

time — life of plants vs. animals and the time it takes for chemical reactions

the four operations — these are used for working with all of the above

We began our project with an exploration of electricity. My motive was premeditated. I understood next to nothing about it. What better way to replace the myth of omnipotent teacher with a true model of perpetual learner? It was risky but it couldn't be more honest. Modeling the struggle was at the heart of teaching them to embrace it. Besides, as part of my PALMS Project I would have the help of Mary Jane Schmitt, my PALMS specialist, who also confessed to understanding little about electricity. We decided it would be fun to risk together.

Mary Jane came in one day with a bag of batteries, wires, and bulbs and posed these challenges: Light the bulb using the materials mentioned above. Try to do it more than one way. Students worked in groups of two or three. They reasoned, argued, proved, and concluded. It was a big hit. They were never more engaged. They did the same thing with the prediction sheets M.J. passed out next. It was another opportunity to conjecture and convince.

Nancy even enjoyed the mental math piece in which M.J. asked how many volts of electricity would six batteries yield? Nancy was spouting out answers to 6x1.5 and 14x1.5 instead of agonizing over what to do with the decimal point. She was doing math and having fun. Everyone succeeded, some even had their first chance to shine...

Nancy was spouting out answers to 6x1.5 and 14x1.5 instead of agonizing over what to do with the decimal point. She was doing math and having fun.

Bob who was skilled in carpentry, plumbing, and wiring was our resident electrical expert. Even Ed Weaver, our visiting chemistry/physics professor from Mt. Holyoke College validated Bob's explanation of A/C vs. D/C. I had learned my first research lesson from Bob—my students had a lot to teach me.

Tammy's journal entry:

We learned today how to light a light bulb by making a complete circuit. Bob taught us that you need direct current (D/C) to make it happen. We also learned some math. If one battery has 1.5 volts and you tape 25 of them together like Bob showed us, to see if the bulb burns brighter (it does) you can figure it out in your head that you are using 37.5 volts of power to do it. You multiply 25x1=25 and then cut that figure in half since .5 means half. 25/2 =12.5. Add it all together and you get 37.5.

The next day Kara and Ana began class with a depiction of what happened when they held a "show and tell" at home with borrowed classroom materials.

Ana: "I asked Salomon (her husband) and my kids to light the bulb. THEY COULDN'T DO IT!!!! It was so cool. I actually could do something that they couldn't." (During class Ana was one of the first to light the bulb unassisted.)

Kara: "Dana (her 6 year old son) was able to light it right away. I was so proud of him. Then he started asking me if other stuff would act like a wire and then he was lighting the bulb with my car keys and earrings. He even took it in to school for

show and tell. The teacher wrote me this note saying that it was the best 'show and tell' they'd ever had. Dana was so proud; all the other kids thought it was so cool!"

Shivers still run down my spine when I remember that day in class. There was a time when I would have questioned whether we had really done any math in those first two days but now I know better. After all what had happened is at the very heart of the NCTM *Standards*. It's called "mathematical power":

"Students should develop mathematical habits of mind...they should be encouraged to explore, to guess and even to make and correct errors ...they should read, write, and discuss mathematics; they should conjecture, test, and build arguments about a conjecture's validity" (p. 5).

We stayed on electricity for four weeks. Kyle noticed something about Ohm's Law (E=IR where E= voltage, I=current, R= resistance) when he was looking up a definition for resistance. That sparked a number of lessons on interpreting equations and operation sense. We even integrated social studies before we were through by reading about Alessandro Volta's and Ben Franklin's early experiments with electricity. Was there any scope and sequence or schedule? Not at all. Inquiry framed the time line. Both mine and theirs. It was truly a case of one thing leading to the next.

It was the students who suggested we move to chemistry next since we'd already begun to explore the elements and their electrons in our electricity unit. They were beginning to recognize the value of connecting old ideas to new...

Nancy: "We'll probably hang onto the stuff we learned about atoms and electrons and protons if we continue to learn about it in real chemistry lessons. God knows I don't want to forget all of this! Aren't there equations like Ohms law? Maybe we can learn more about equations in chemistry?"

We spent three weeks on chemistry but it wasn't as smooth as electricity. We ended up working a lot on balancing chemical equations with the help of toothpicks, marshmallows, and colored chalk to represent different atoms and molecules. (A true exercise in mathematical reasoning as well as number and operation sense.) The class really struggled with it. When I suggested that we move on to something else the most wonderful thing happened—They refused! What a measure of success. They were learning to value the struggle. From my journal:

When I suggested that we move on to something else the most wonderful thing happened—They refused! What a measure of success. They were learning to value the struggle.

> ***In class today during our second research discussion the students took the reins from my hands. It happened when Tammy said, "Maybe it [chemistry] is really hard and maybe you could have presented it better but like you keep telling us making mistakes is part of it. Why don't you try it a different way and we'll work even harder at trying to get it." It kept on going when Ana said, "Won't it feel good if we get? You'll be so proud of us and I'll understand something Salomon doesn't! He can't believe I'm learning chemistry. He's got his GED and doesn't understand any of the homework I'm bringing home."***

In the end everybody but Bob, Nancy, and Donna expressed confidence in their ability to balance chemical equations and successfully (75%) completed the final worksheet.

FINDINGS

During the second and final research meetings the answers to the questions of what is math and science had changed dramatically. These changes reflected not only an expanded view of math and science but expanded thinking as well. I believe the richer, sometimes metaphoric, definitions are evidence of this greater complexity of thought. Additionally, the students' definitions contained the air of confidence and sense of subject appreciation lacking in their earlier ones. Math and science were, for the most part, no longer merely interesting amusements or boring tortures but valuable tools for understanding our world. I feel, strongly, that this growing appreciation for the subjects' utility was a direct outcome of marrying our study of math with science.

Mario: Math is just like riding a bike, you never learn it the first lesson, you have to keep trying no matter how many scrapes and bruises you have. Just like a bike math takes you places.

Donna: Math is a world of numbers and more than that. It's reading, thinking, and a lot of patience.

Tammy: Science is like math because like the universe we understand it a little at a time.

Kyle: Science is a way of looking at the world. It's based on fact and grows through theory. I think math is a science too isn't it?

Carol: Math is more than I thought it was. It's not just numbers and operations. It's thinking about them and where you use them.

Ana: Science? I still agree with what Mario said about science in the first meeting. It's boring but at least I can finally understand it. I know a lot more than my husband and that makes it worth all the thinking I have to do.

Bob: Math is very difficult. One of these days I'm going to get it. The numbers are what I see because they're what give me the trouble. The thinking part I understand.

Nancy: Math is a subject conjured up by demons to drive people like me crazy. Actually, I'm beginning to see that it's just a very logical way of thinking about more than numbers. Sometimes I can do it and other times I can't. That's the part I hate most.

Karen: The definitions of math and science are now so blurred for me I'm not sure where they part company. Math IS a science. Both train our minds to think critically as we view the world. Both ask us to search for answers while providing us with the tools to do so. I know one thing...they're both fascinating!

At this moment I can't help but wonder why this obvious union of math and science was so obscure for me at the onset of the project. I had honestly questioned the number of connections possible! (Those puzzled looks from some of my colleagues suddenly make sense.) Even the *NCTM Standards* note the apparent connection:

> ***"Many opportunities to show the connections between math and other disciplines are missed in school. Math arises not only in science but in other disciplines as well."*** (p. 86)

Could it be that the answer for my own ignorance lies in that same quote? As a student, had my teachers missed out on the many opportunities for connections? After all I was an honor student who loved science. Maybe it was my parochial vs. public education or my distaste for math that was to blame. I'm worried, however, that my first instinct is the right one. Like so many students herded from teacher to teacher I was taught subject after subject in isolation. Perhaps because I was never asked to see the connections I never did. How many of us, I wonder, are still doing the same, especially those insufficiently trained in math?

From the *NCTM Standards*:

> ***"Students should have many opportunities to observe the interaction of mathematics with other subjects...To accomplish this, math teachers must seek and gain the active participation of teachers of other disciplines in exploring mathematical ideas through problems that arise in their classes (p. 84)."***

In May we had entered the world of astronomy. We were still working on scaling down galactic distances when we stopped maintaining our journals and started to forget that we were doing research. We were too busy experiencing learning at its best. My greatest assessment tool ended up being a video project M.J. got us into as part of PALMS. Under bright lights with cameras in our faces we were scaling down the Solar System, the Milky Way, and the distances within. Calculators were buzzing, zeros were flying, and mistakes were being made comfortably. In the video I had really faded into the distance. As I viewed it for the first time I knew that it was the proof of my success. Math wasn't lifeless. It was not teacher driven. It was dynamic for us all.

But what has any of this got to do with preparing for the GED? A lot! In late June, during the last week of school, Linda, Nancy, Donna, Kara, and Tammy came to me (collectively) to say they had decided to take the GED together at Holyoke Community College. In August each of them called me at home with their good news:

Linda: passed GED with 270 - math 42; science 59

Nancy: passed GED with 232 - math 40; science 43

Donna: passed GED with 243 - math 50; science 46

Kara: passed GED with 235 - math 52; science 41

Tammy: passed 4/5 of the GED; will retake writing this fall; math 46; science 44

Now, as I look back on my means of teacher assessment (journals with written summarizations, discussion and observation) I no longer feel insecure about their reliability since the GED scores seemed to corroborate my own findings: In math the students had grown in reasoning ability and problem solving and had increased their strategy repertory. In science, they had gained added confidence in their abilities to understand this vocabulary laden subject matter. They had true background knowledge in chemistry, astronomy, and the physics of electricity, and could sort out necessary from unnecessary details and read more critically. Students even reported feeling comfortable with the passages on astronomy and chemistry that, luckily, were on the test.

Even more noteworthy were those students who had chosen *not* to take the test. At the beginning of the year Carol, Kyle, Ana, and Bob were like many adult education students in search of the fast track to a GED. In June, however, they sought something more — to pass the test with what Bob called a "respectable" score. As Carol said, "Most of all I want to feel smart not just lucky that I made it. I'm not ready yet but I will be...Next year I know I will be."

At the beginning of the year Carol, Kyle, Ana, and Bob were like many adult education students in search of the fast track to a GED. In June, however, they sought something more.

As a teacher, the attitudes of these four students were more valuable than four passing scores. I admire them tremendously.

The connections they saw at the end filled two sides of a big white sheet. Here's a few:

distance — outerspace and light years and word problems with D=RT

measurement — metric and standard American used in chem-labs and at home for tons of things (inches to amu's)

geometry — Greek mathematicians used this for scientific knowledge for figuring the earth's circumference; sailors use it with stars; useful in carpentry

scaling — used for map making and understanding distances in space; need to be good at calculators and large # division

equations — to represent compounds & reactions in chemistry and operations of variables in algebra

From Kyle's journal during our astronomy unit: Today I learned that the numbers, which would have overwhelmed me a few months ago, are mere fractions compared to the vastness of our universe. The number 0 no longer seems so insignificant.

Ana's second answer to what is math sums it up best:

Math is like a child learning to walk—once you get the hang of it the rest will fall in place. You need math to survive like you need food... Math is like life itself—complicated! When I do math I am in another dimension lost in space. I always find my way back though. I will never give up.

EPILOGUE

Remember the Wizard of Oz? At last I understand the last scene between Dorothy and the Good Witch. Once Dorothy realized the Good Witch had known all along how to help her return home she asks (as I have). "Why didn't you simply tell me?" The witch replies: “You wouldn't have believed me; you had to find it out for yourself.” For me, teaching to the *Standards* has been a lot like what Dorothy went through. I didn't believe it until I experienced it for myself. The difference between Dorothy and me is that she wants to go back to Kansas while I plan to stay in the Emerald City. You learn a lot about yourself here. Just look at what it did for the Cowardly Lion.

"IT'S NOT THROUGH THE BOOK"

By Leonora E. Thomas, New Bedford Adult Education Program

The traditional way of teaching a pre-ASE or ADP math class is to:

- ✔ Teach content in a linear method (whole number, fraction, decimal, percent);
- ✔ Depend upon paper, pencil, and book (workbook and worksheet);
- ✔ Teach isolated computational skills with word problems as their only application; and,
- ✔ Assess students by means of a standardized program of developed tests.

> ***"In the adult basic education classroom, curriculum design must include approaches to making mathematical connections which allow the learner to explore problems using appropriate technology and describe results using a variety of mathematical models or representations including graphs, concrete, verbal, and algebraic models or representations."***
>
> MASSACHUSETTS ADULT BASIC EDUCATION MATH STANDARDS, STANDARD 4: MAKING MATHEMATICAL CONNECTIONS

I wondered what would happen if, once a week, I ran a math class where:

- ✔ Math skills were taught in a non-sequential manner;
- ✔ The lessons incorporated *realia*, manipulation, and technology; e.g., calculators and computers;
- ✔ The lessons were derived from specific questions posed by students, as well as topics known to be of interest/need to this adult group; and,
- ✔ Assessment was derived through interviews and group discussions of what was learned.

I strongly believe that adult learners are being held back today not only by a lack of self-esteem and self-confidence and simply just trying to survive in their daily lives but also by the constraints of language, capability, and the educational system itself. The insistence of a linear progression in mathematics, where algorithms in books are repeatedly done and must be "mastered" before moving on, cannot continue.

Having had an ESL learner for a father, I can tell you that the system failed not only him but many others who came to learning centers hoping to advance their

English and math skills by making them do repetitious, tedious work in the same content area and in the same way, until they succumbed to frustration and boredom at not progressing.

I remember my father asking me, while I was still in grammar school, to teach him to read because he wasn't learning anything attending the night sessions in adult education. While the teacher was still telling him to walk to the board and pick up the chalk, he was communicating faster in English with his co-workers in the factory! His teacher also did not think it was appropriate for him to learn arithmetic since he hadn't "completed" his ESL classes! Of course, it didn't matter that this was a real person, with a real family working in the real world and needing real life skills! He hadn't completed his English classes!!!

As I grew up, and being of Portuguese descent, I saw many more situations like ours happening all around me and entered my teaching career with a lot more awareness than some.

Adults come to us with varying degrees of knowledge and skills already attained in the outside world, and many already have some understanding as to where their needs must be met and developed. They are looking to us, their teachers, as their last hope in helping them to attain a better life by providing them with those basic skills they somehow never really mastered or never perhaps even learned in their educational development.

I cannot think of any better method of teaching than by letting the students learn for themselves... by questioning, by doing, by sharing, by thinking, and finally, by wondering. This has always been my premise for teaching, and this year, when my involvement as one of only six statewide Adult Education representatives in Project PALMS led me into an association and working relationship with the Massachusetts ABE Math Team, I soon found myself in the middle of proving my conjecture.

I cannot think of any better method of teaching than by letting the students learn for themselves... by questioning, by doing, by sharing, by thinking, and finally, by wondering.

Through this statewide connection, I was given *carte blanche* to implement and develop on a larger scale not only what I truly believed could work but that which I was already doing on a smaller scale. I needed to further question myself and develop and document my results. My explorations were time consuming and, at times, exhausting. However, I feel that my work with the ABE Math Team and the experiential knowledge gained from the year's labor have greatly enhanced my teaching capabilities and benefitted my learners.

CONTEXT

I teach a pre-ASE (pre-GED) class at the New Bedford Adult Learning Center which is part of the New Bedford Public School System in New Bedford, Massachusetts. We are currently located on the third floor of a bilingual pre-school where the classrooms have been carved out of an old auditorium. Our classroom is small and windowless but bright, comfortable, and quite welcoming.

The auditorium itself is drab with poor lighting and very high, water-damaged ceilings. Although the classrooms have had dropped ceilings installed, with some tiles left "open" for "fresh air", the acoustical situation remains horrendous due to the fact that this floor was built to be an auditorium. Many times I've caught my students trying to cover their ears in order to concentrate on their reading or on a test. They've had to ask me questions repeatedly because they were unable to hear either me or another student speaking. Even taping my class has been, at times, difficult because of this noise level.

Another problem at the center is the bathroom situation. Students must go down to the first floor in order to use the one ladies' or one men's room. This has been difficult with adult ed. teachers and adult ed. students, as well as the pre-school's own faculty, ALL using the same facility!

The learners are required to attend classes daily, Monday through Friday, from 9:00 a.m. to 11:30 a.m. Some of them have worked third shift, others are on second shift while yet others don't work at all. These are either currently unemployed, on disability, or are homemakers. The program requires students to divide class time between language and math to focus on their preparation for Adult Diploma or GED programs. The tasks within that structure are set up in linear fashion.

The men and women in my class this year were mostly American citizens by birth or naturalization. They were also mostly of Portuguese descent while one was from Vietnam, one was from Puerto Rico, and two others were Dominican.

Two of the students have been in our program for several years, moving up slowly through the ESL program. I was interested in seeing if my method could work with them in order to speed up their learning process since I could see that they were getting frustrated by their long haul. Most of the students were working with English as their second language. This was of concern to me with comprehension and application of mathematical concepts. Age and ability were also factors I had of concern when beginning my endeavor.

I was interested in seeing if my method could work with them in order to speed up their learning process since I could see that they were getting frustrated by their long haul.

The students in my class sat around tables which I arranged in a circle. Here they shared not only their tables but also their ideas and their homemade goodies. They really were eager to learn and go on with their lives. They encouraged each other and truly wanted to help one another. There was a cooperative effort going on, to learn as much as they could, as fast as they could, in order to get their high school diplomas! They were highly motivated students who continued to return to my Tuesday math classes even after moving into the Adult Diploma Program to complete their studies.

METHOD

In order to answer my questions, I devised several lessons incorporating both language and reading with mathematics. The "mini-units", as I called them, would usually begin with a reading or language lesson. These were mainly articles I had found in periodicals or newspapers to be of interest to the students. They were also educational and a good basis for incorporating a math lesson. The goal of each "mini-unit" was to teach algorithms by connecting them to something in real life or everyday practice.

The articles I chose to begin our lessons varied in composition. We read about the average sleep requirements of Americans over thirty, and overall by age groups; nutrition and percentage of fat calories in foods; the amount and types of trash produced yearly by each individual in America; the how's and why's of homeless Americans; and about the whaling history of New Bedford, to name just a few.

The topics always involved new vocabulary, which was discussed both in and out of context and sometimes researched as well. Question and answer periods followed the readings (which were always done by the students) in order to address any problems in comprehension the students might have. *All* of the above were necessary, particularly for the ESL students, since one of the greatest barriers to learning mathematics is language; and for ESL students, that problem is compounded by vocabulary and idioms.

Once we had a topic, and understood its implications, we carried it into math. The students could count on having a whole group math session every Tuesday. During the remaining part of the week, the students would continue in their individualized linear progression of mathematics due to time restraints and school requirements.

I tape recorded in order to catch some of the students' questions, problem areas, and particular "eye-openers" which I may have missed or wanted to recall. Photos were taken as we went along to capture our "mathematical moments." Manipulatives of all types were used for hands-on learning.

The students worked mostly in teams of two or three, and at other times as part of a larger group. They needed to cooperate with each other in order to learn and were responsible for their partner's ability to comprehend the subject matter before taking the next step.

My method of assessment was also easier than anticipated because of the many opportunities I had for observation and discussion. I could see what the learners really knew or weren't quite yet grasping, if they really were learning. Whereas, if they did an algorithm repeatedly, I wasn't sure if they truly understood what they were doing or even its application.

The following lessons I describe will illustrate three typical levels of connections: connections between content areas, connections between content areas and math, and the connections for learners between what they knew and the new ideas/material which I presented.

CALCULATOR POWER

First I gave everyone a few lessons on the use of the calculator. The students were allowed to "play" with their calculator and tell me what it could and couldn't do. We discussed how it could help them in learning math. "How can I figure sales tax on here?" asked Ricki. "Yeah, and suppose I hit a wrong number? Do I have to start all over again?" asked Maria. "I always wanted to learn how to use my son's calculator," said Georgia. "So do I," said Teresa, "But I'm afraid!"

Since I had multi-level math students, and the use of calculators was part of my question, I was determined to incorporate them in every lesson and encouraged my students to use them as often as possible in order to change numbers into percents, etc. Whether the adult learners were in long division or percents, calculators would allow all students to participate in problem solving on an equal basis. My reasoning was that if students were working on a particular algorithm and either knew or were learning the mathematical process, the calculator would facilitate the computation not only for the quicker student but also for slower learner and those who continually made careless errors. I also tried to make them feel comfortable in using memory. "I always wanted to learn how to use that!" said Victor rather excitedly. "Me, too!" said Alfredo, "Especially since my son can, and I can't!"

> ***My reasoning was that if students were working on a particular algorithm and either knew or were learning the mathematical process, the calculator would facilitate the computation not only for the quicker student but also for slower learner and those who continually made careless errors.***

We practiced the use of memory and other calculator skills in our "Food Cents" mini-unit. Since I knew what my class needed to improve their estimation skills, I incorporated some work on grocery receipts and unit pricing. Here students were required to estimate then correctly tabulate the food items, taxable items, and coupon deductions on their receipts. They practiced rounding numbers, grouping items, entering and subtracting from memory, and found out just where that "extra penny" really went on an item marked 3/$1.00... "I never knew it was right away on the first (item) in the group," said Ricki. This work not only helped with their dexterity in manipulating the calculator and in reinforcing some basic arithmetic, but also provided them *immediately* with some very useful skills!

GRAPHING POWER

We began our next "mini-unit" by reading an article in *Today's Times* on insomnia. Then we studied the charts contained in it on American sleep habits and worked on interpreting their graphical representations. At this point students were asking, "What's the difference between the graphs?" and "Are there any other types?" also, "How do we know which one to use?"

Since I already knew that they had difficulty interpreting line, bar, and circle graphs from their pre-tests, I decided this would be the best time to teach graphing. This conflicted with the traditional mode of teaching whereby math is taught in a step by step linear progression; i.e., whole numbers, fractions, decimals, and percents with graphing following.

The learners' inquiries into the formation of graphs led to more work on their development. They were instructed to bring in graphs of interest to them. We discussed not only the vocabulary for, and the types of, graphs brought in, but also proceeded to read and interpret more and more of a variety of them. There were bar graphs on everything from the consumption of candy bars per person per year, to circle and bar graphs on the concentration of wealth in the United States, the middle class income levels, the percentage change in the distribution of family income, and most important, how the less educated were suffering more. This last one had a great impact upon my learners. Here they noted the importance of a high school diploma and its *connection* to a better income!

Next we incorporated work on ratios and proportions and began graphing packages of M&M's by color and content. This work gave them the framework needed to begin formulating their own graphs including, eventually, a human circle graph representing each individual's amount of garbage, and later on, a graph on the family budget.

NUTRITION AND PERCENT FAT CALORIES

One of our most important "mini-units" began with an article we had read on food labels and how they were changing to concentrate on better diet and nutrition. The article contained a wealth of vocabulary terms and led us to another article on vegetarianism. "Just what is a vegetarian?" I was asked. "That's people who don't like meat," said Teresa. "All meat! . . . What about chicken and fish?" asked Victor. "I think some do and some don't," said Ken.

We clarified this when we read about the three different types of vegetarians and how to differentiate them. We also read about the pros and cons of becoming one from the viewpoints of both a vegetarian and a non-vegetarian.

> ***We read about the pros and cons of becoming one from the viewpoints of both a vegetarian and a non-vegetarian.***

The article discussed the food habits, tips for transition, reasons for not eating meat, and myths about vegetarians. At this point, the majority of students agreed that they could never even think about becoming a vegetarian. "They're too strict in their diet," said Serena. "I could never eat that way!" exclaimed Maria.

Then it was time to "rethink our plates" by reading two pie charts. One contained the typical American diet for lunch, the other, a typical vegetarian lunch. It depicted fats, protein, and carbohydrates in percentage as well as the total amount of calories, cholesterol, and sodium for each. Using the calculator, students calculated the amount of protein calories by multiplying the percent of protein by total calories. Their work on graphs had made it easier for them to make decisions on whether to use multiplication, division, etc. They did this for each of the carbohydrates, fats, and proteins in the charts and couldn't believe the "fattening" results. They also noted that the vegetarians had more food to eat, less calories, and zero cholesterol for lunch!

From this stepping stone of awareness, we did our own investigations. I had asked students to bring in their empty boxes and labels from cans showing the nutritional content of each product. We had enough contributions whereby each student could be responsible for working on two items. Then they read the nutritional content of their products, including sodium levels and grams of fat, and discussed which products they felt were more nutritious using the information given, the percentage of fat calories in each product. Next, they were given a formula to calculate the percentage of fat calories in each product:

"Rule of 9"

$$\frac{\text{(number of grams of fat) X 9}}{\text{(amount of calories per serving)}} \times 100 = \%\text{ fat calories}$$

By following this "rule of nine", where one gram of fat equals nine calories, the students calculated the percentage of calories from fat in each product and then compared the results to their original decisions on the best products to eat. The new results were astounding to the students, particularly in the areas of so- called "low-fat" and "dietetic" foods.

Finally we talked about the daily nutritional guidelines in percentages of calories to maintain a healthy diet without becoming a vegetarian. I then left the class on their own to complete a large graph for the classroom representing the percentage of fat calories per product examined. Normally a teacher would stay and facilitate or guide the students in drafting the chart. However, I felt that my learners could really handle this on their own because of their practice in graphing. Moreover, I thought this would be a good assessment of all the skills they had been acquiring, including cooperative learning, as well as a good assessment of their ability to put on paper what they were really learning. The result, as it turned out, was a magnificent and colorful piece of work produced by the entire group attesting to their joint effort and abilities!

GRAMS VS. OUNCES

When students posed the question, "What's the difference between a gram and an ounce?" and "Which one's heavier?" we rolled into a discovery session involving grams.

Here the learners examined the various plastic gram weights and then decided to weigh and compare a list of objects which they had picked up from around the room. The group huddled around a table set up with a balance, gram weights, and the various objects chosen such as a button, a feather, a pencil, a ruler, etc. I asked them to estimate the weight of each object by placing it in their hands first, then find the actual weight of the same object in grams before moving on to another object. Students would take turns dropping in an item and balancing the scale while everyone watched and recorded the results.

At first I led them through their discovery, asking questions, commenting, trying to get them to think, make conclusions, and so on. Later I chose to observe and listen more to their reasoning and allow them time to work and agree or disagree with each other. They discussed and recorded their observations on paper. Responses to the "feel" of the one-gram disc-shaped weight were, "It feels like a feather!" and "It feels like a potato chip!"

The students continued this way until all the objects had been weighed and everyone had a chance to manipulate the scale and weights. The learners compared their estimations and calculations of each object and discussed the reasons for any discrepancies. Later we continued our explorations and observations using dried beans, rice, and sugar. They left this session still talking and wondering. "What if . . . ?" and "How come....?"

WHALING HISTORY OF NEW BEDFORD

Our last group lesson came at the end of the school year when my learners began to discuss the problems of their city, New Bedford, and seemingly had very little pride with respect to where they were now living. I talked with them about my memories of it when I was a child growing up there and how beautiful the same city was then. Next I gave them some background information on its historical significance. This opened up a new "world" to them, one in which they were extremely interested in discovering. It whetted their appetites for more about their once famous and prestigious city!

We not only read about and discussed New Bedford history but a docent from the New Bedford Whaling Museum, who I invited to our class, gave us a very informative lecture and slide presentation on the people and customs of the whaling era of New Bedford. He also brought in whaling artifacts such as whale oil and spermaceti candles, a toggle harpoon, baleen, a handmade rope the length of a sperm whale, as well as several other items of interest, and discussed whaling vocabulary and history with our learners. The students were impressed by the extension of the rope in the building which helped them to experience the great size of the mammal.

In the follow-up math session, the class was given a review vocabulary sheet of whaling terms as well as a copy of a sliced full-length view of a whale ship. Its decks, full cargo, and living quarters were outlined and labeled. After being given some mathematical figures, the students determined the size of a whaling ship and its crew, its ratio compared to the whale, and the amount of oil carried by the ship in casks, as well as its worth, and the net pay of each man on the ship. This was followed by a walking tour of a Historic New Bedford and a remarkable visit to the Whaling Museum.

FINDINGS

With each "mini-unit" I taught, I found that if the lessons were meaningful and interesting, the students were more eager to learn and more willing to attend. In fact, attendance at class, particularly at Tuesday's math class, was never a problem.

"This is a good way of teaching. You can see it better this way." said Ricki. "What I like about this class," Alfredo said, "Is that it's not through the book; it's more physical!"

As I progressed, it became clearer to me that students could definitely be taught math skills "non-sequentially"; that is, taught independently of traditional structured ABE curriculum. I found that it wasn't necessary for the learner to know the various computational phases of fractions or decimals in order to be able to do percentages, particularly on a calculator, nor did the learner have to wait until he or she had progressed through the various stages of basic math before understanding ratio, proportion, graphing, and measurement. Rather, these skills could be taught independently of the learner's previous mathematical experiences.

Two of my students, for example, Georgia and Serena, were only completing long division skills in Contemporary's *Number Power One* (Contemporary) yet they were already figuring area and perimeter, ratio and proportion, graphs, measurement, and percentage with the aid of the calculator, and were, therefore, keeping up with the rest of their group! Each knew the method for figuring out problems being discussed in class but were initially being held back because they never progressed beyond the repetition of whole number algorithms. Georgia, especially, had felt very intimidated by mathematics when she first began. She said that she not only "needed to take her time learning" but also that she "really wanted to take the time to understand it first."

Georgia could tell us that certain numbers needed to be divided in order to obtain an answer. Her only difficulty came in interpreting that answer. For example, if she had a ratio or fractional amount that needed to be converted into a percent, she would get a decimal and need an explanation on how to change that to a percent. When we discussed with the class the reasoning behind moving the decimal point two places to the right, several of the students', as well as Georgia's eyes, brightened! "So ... ," she said, "A quarter is 1/4 as a fraction, and its decimal is .25, like money, and... that's a quarter of 100, so ... it's 25%. They're all really the same thing ... I get it! That's so ... interesting!" she would say.

"So ... interesting" became Georgia's favorite words. This 40-year-old mother of two who had never studied beyond third-grade arithmetic was soon grasping the concepts and getting the solutions quicker than many of the other students. What's ironic is that she considered herself to be "slow."

Georgia had many of these mathematical "Aha's!". When interviewed by a news reporter doing a piece on adult learning, Georgia was quoted as saying, "I've learned so much here. You don't realize just how far ahead you are!"

She was always a lot further than she thought. Her conceptual ability, coupled with the use of the calculator, had truly enabled her to reach!

Another student, Vincente, was a 21-year-old high school dropout trying desperately to obtain a high school diploma in order to get a job and help support his baby daughter. At first, Vincente lacked self-esteem and self-confidence. When I met him, he said quietly, "You know, Lee, sometimes I feel like a dummy 'cause I haven't got nowhere and I have a hard time reading and my math is lousy ... and now I got a kid!"

He could sight read pretty well but really had a poor grasp of phonics and grammar. Both his spelling and his math were extremely weak. The students weren't sure of him either, due to his biker look and tatoos on both his arms and hands. But through our cooperative learning classes, he soon became one of the group.

Respect for Vincente's thinking came about during our "gram vs. ounce" lesson. Again, the lesson had been derived from questions that had been posed by the learners. "What is the difference between a gram and an ounce? Which one is heavier?" they asked.

Through various estimations and calculations of the objects, the learners explored the concept of gram and weight. But when the class had to estimate the weight of a twelve-inch plastic ruler in grams, the majority of the students had answers that were very much off. The average weight that had been estimated was five grams when, in reality, the actual weight was 15 grams! The class couldn't understand how they underestimated its weight. Statements were made like "It doesn't feel that heavy!" and "I can't believe I was that far off!"

As they tried to go on to another object, Georgia suddenly burst out with "I can't believe that ruler was 15 grams, though!" "I know . . . me either," said another. Teresa bemoaned, "Why is it so heavy if it doesn't look or feel that heavy?" Another student, John, took the ruler and stretched it out lengthwise over the palm of his hand, fingers to wrist. "I know why," he said, "Because if you put that part in your hands in this way, this part weighs more with the ruler." What he was trying to say was that it felt heavier lengthwise than across the palm of his hand. Another student, Victor, was trying to stand it in his hand and with the students following his modeling, he stated, "You can hardly feel any weight this way!" Everyone agreed and was surprised. I was excited that they couldn't let go of this problem!

In the background, I could hear Vincente's explanation for the discrepancy as he spoke to another student. He was telling Ken, "The one-gram mass is about the same size and thickness as an inch on our plastic ruler. If you put 15 of them (the one-gram masses) all lined up, that would look like a ruler (side by side) so, they'd be the same . . . 'cause that's plastic and this is plastic. You can see it. It would take at least twelve of them." I had him repeat this to the entire group. "Oh... ," I heard, "Eh, Vincente, WOW!" said Alfredo. "Great!" said another.

I complimented Vincente, who at this point was smiling quite confidently! He was thinking and using manipulatives to his advantage. Here was deductive reasoning, logical thinking being imparted by a strange looking fellow whom everyone had been unsure of since day one! The students now saw him in a different light and not only continued to ask him more questions later on, but also valued his opinions as we continued in each lesson. Vincente also made use of the calculator whenever possible for computation and progressed more rapidly with it. In fact, he was the first student (working linearly) in class to begin the Geometry book!

We continued to work on areas of interest and questions derived from those topics throughout the second term. These would invariably lead to more questions by the students and provide a learning continuity through connected subject matter. For example, after completing the first part of our gram vs. ounce class, Maria stated, "I still don't know the difference between a gram and an ounce!" This extended our explorations into another class day! Also, Alfredo was so impressed with our studies on the whale that he posed further questions on the lung capacity of whales versus their relationship to human lungs. The docent from the Whaling Museum (who was also a volunteer tutor for my math class) and I were so impressed by his curiosity that we felt compelled to follow through on his questioning by developing a few more word problems in that area.

Alfredo was so impressed with our studies on the whale that he posed further questions on the lung capacity of whales versus their relationship to human lungs.

By the time we had reached our "Mapping and Miles Per Gallon" unit in late spring, I found that new students entering my class lacked the same degree of skill as my regular students in estimation, logic, and reasoning. The new students' estimation skills were always way off by comparison, and my regular class knew it! Through practice, the estimation and calculator skills of my steady group had been improving and it was becoming more obvious to them, as well as to myself.

The learners had made their own budgets, graphed the country's garbage in the form of a human circle graph, and compiled a neat and colorful graph representing percentage fat calories of the products they were consuming. They had measured the area of the classroom for new carpeting and the walls for painting, as well as the cost for each job. They had learned how to take a trip cross-country while I had learned to take a back seat to their learning experiences. By the end of the school year, I found that I had learned a lot from my students through observation, questioning, and allowing them the opportunity to "put it together" by themselves.

I found the best method of assessing was to ask questions of the learners both during and after the learning process. I questioned them within their workgroups in order to get more individual responses and to see if the cooperative learning process was working. I found that having the students work together enabled *ALL* the students to grasp the problem better. They shared their ideas better with their peers because they were not intimidated by the "All-Knowing" teacher. They also began to see different methods of solving the same problem which truly enriched their learning experience.

Students gained a better sense of this learning experience when I allowed them to explore and manipulate. I allowed them the space to research and develop their own graphs without my interference and to visualize problems whenever possible.

The use of manipulatives and calculators enhanced the learning process. In one "mini-unit" we incorporated a trip-tik (i.e., a detailed map from a travel agent with the most direct route to your destination outlined) that had been cut up into three sections along with a map for our "Cross Country Trip." Here the class was divided into three teams of students who worked frantically on each of the three sections calculating mileage, distance, cost of travel, and time, while learning geography, map reading, and identification of symbols, etc. When I laid out twelve-inch rulers in the middle of the classroom floor to represent a square foot, and then several more rulers in square feet to make a square yard, the students were able to visually grasp perimeter and area a lot easier. "Now I understand what they mean by a 'square' foot," Ricki said. The students also began to manipulate all the rulers and yardsticks we had in the classroom for

When I laid out twelve-inch rulers in the middle of the classroom floor to represent a square foot, and then several more rulers in square feet to make a square yard, the students were able to visually grasp perimeter and area a lot easier. "Now I understand what they mean by a 'square' foot," Ricki said.

further study, themselves, and again for our "Think Spring" unit on area, perimeter, and job cost, as well. The use of trip-tiks, rulers, gram weights, newspapers, even balls of string, all enabled the learners to "see" as well as "do."

The calculators freed them from tedious computation, some of which they could not have done yet with paper and pencil or at times correctly. I found that the barriers of ability, age, and second language all contributed to computational error and that the calculator facilitated these processes for them. Maria and Teresa were both ESL students hindered by all of the above learning factors. They took a lot longer to process material and made numerous computational errors even though they knew the correct method of solving the problem. By using the calculator, these students, along with others such as Serena and Georgia who were either in division, fractions, or decimals, were able to reach the same solutions as their classmates in nearly the same amount of time. Because they were able to work with the calculator, they were not hampered in working with the group by their linear movement in math.

Cooperative learning had flourished, and group dynamics had proliferated. People were leaving class wondering "What if? . . . Why? . . . Suppose? . . ." and looking forward to "our next math session."

Calculator skills had improved with use, had facilitated problem solving, and had empowered my students with success. Given time, the students not only did their own estimations but began to realize what was reasonable versus what was "off."

I found through the different units, students were covering the same estimation practices, the same calculator practices, the same repetitive skills that are a part of the everyday classroom.

CONCLUSIONS

In conclusion, I believe I have found a basis for teachers to re-think their classroom in terms of how they teach their students. For the adult learner who comes to us highly motivated, it is important to reinforce that motivation through concrete and highly meaningful learning experiences. There are so many life-change events prompting them to seek new learning opportunities. If we cannot help them crystallize and discover, then we cannot truly hold them. Working linearly, algorithm after algorithm, with no real application is not teaching! Teaching an algorithm in terms of its application to relevant situations is! Adults need to be able to integrate what they are learning with what they already know!

Calculators are an invaluable tool in the classroom. Adults can compensate for being slower and can make fewer errors in computation as well as function better in the real world by knowing how to use one. It empowers them with capabilities they may not have using paper and pencil alone. They will take more risks in finding solutions to problems and can build their self-confidence and self-esteem, something many of them need when they first re-enter the classroom.

Adults can compensate for being slower and can make fewer errors in computation as well as function better in the real world by knowing how to use one.

The use of computers for reinforcement of skills, as well as word processing language, is also very important to the learner. Familiarity with them, as well as other manipulatives such as weights in measure, trip-tiks, maps, globes, etc., are all a part of their real world and enable them to see it in perspective.

We should allow the needs of our students to be the focus of classroom learning. Their questions are the best source for lessons anyone can devise!

Lastly, assessment of students and what they have really achieved through learning is best done through questioning *during* that learning process when the information is being gathered, sorted, and processed by the individual - also, *at the end*, when the learner can give you the results of what he/she has accomplished. The current assessments are outdated and in no way truly reflect conceptual knowledge gained by our students. Rather, they reflect rote memorization of skills from books.

This summer, to follow up on these precepts, I will teach algebra to my pre-ASE class regardless of what areas of arithmetic they are in. I want to see if someone who hasn't really mastered long division with paper and pencil, for example, can work on algebra. I think I already know my answer!

Through my research I have already found that making *CONNECTIONS* truly *WORKS*. What I have concluded is that it really makes the whole learning process a lot more palatable!

EXPLORING THE METRIC SYSTEM IN A WORKPLACE EDUCATION CLASS

by Judith Sulzbach, Quinsigamond Community College, Worcester

The question that I chose for my project is "Will learning about and using linear metric system units give workers another means of discovering and communicating math in the workplace?" The metric system was always a part of the math curriculum at the factory where I teach. The factory is owned by a French conglomerate and sells some of its products internationally. It seemed logical to me that these workers should have a basic knowledge of metric units. However, as I delved more deeply into this, I discovered that the metric system was not used by the workers as extensively as I had originally thought. My students used various charts that had conversions on them, but even then, they did not seem to have a clue as to the relationships within these conversions.

> ***"For the workplace education student, instruction must assist the learner in seeing the connection between problem solving strategies used in the classroom and math-related tasks on the job."***
>
> MASSACHUSETTS ADULT BASIC EDUCATION MATH STANDARDS, STANDARD 4: MAKING MATHEMATICAL CONNECTIONS

As I interviewed a plant supervisor at the company, I discovered that many supervisors must be ISO (International Standards Organization) certified in the metric and English measurement. ISO is a government regulatory agency that sets standards for companies that deal heavily with international countries. Many of this company's parts are made in Europe, and since the metric system standards are used practically world-wide, I assumed company personnel should be knowledgeable in what the basic metric units mean. In talking with different plant supervisors, some said that finding replacement parts for machines had become a problem, because U.S. parts won't fit European-made machines and vice-versa. The ISO certification states that when international companies deal with each other, they must make their respective products with the same specifications as their foreign counterparts. Inspectors and supervisors at this company must take the ISO certification course periodically to keep current on all production manufacturing standards applicable between the U.S., Europe and other international countries.

The metric system curriculum that we taught at my company was very generic, and I knew that I had to adapt my student various jobs to this curriculum to make it seem more relevant and job-related. I read in the Massachusetts Adult Basic Education Math Standards on Measurement, that..."some learners have difficulty in selecting and determining appropriate tools of measurement. Teaching should focus on the measurement systems necessary to workplace needs."

As I got into the teaching and the research of metrics for this particular plant, my question began to change as three themes seemed to impact it. Those themes that began to influence my original question were:

✔ the relevancy of the math curriculum already in place;

✔ the students' lack of motivation due to changes in their working conditions; and

✔ my learning more about how and where metric measurement was actually used in this factory.

As I pursued research into the metric system, I discovered that this system seems to be more orderly, logical and simplistic than the standard one with which we are familiar. I saw an interrelationship between metric units that is advantageous to a learner. Anytime a learner can see a connection between various parts has to be helpful in solving the overall puzzle. To prove the point of how much simpler metrics seems, consider the following comparison:

In the metric system, 1000 cubic meters of water has a capacity (volume) of 1 liter and a mass (weight) of 1 kilogram. Compare that to our standard system where 1000 cubic inches of water has capacity of 554.11 fluid ounces and a mass of 578.02 avoirdupois ounces. Doesn't the metrics system seem simpler by this comparison alone? I know that this is one of the things that I stressed in this project: the importance of pulling something together, crystallizing it to make sense. The ABE Math Standard on Connections came immediately to mind. Making connections between my students' number knowledge and the applications of that knowledge to the workplace will more likely help learners find solutions.

CONTEXT

The factory where I teach math has been the major employer in the town for many years. This plant manufactures abrasives, mainly wheels, for such buyers as car manufacturers, tool makers, and engine plants that make crank and cam shaft grinders and bearings. Several generations in a family might have worked at this facility during the course of their lives. An employee under the old owners might have worked on one job or machine for the course of his employability. Wages and benefits were good; at one time a worker was paid according to the piece goods he produced. This facility has recently been bought by a French company, and among the changes that have been enacted are cutting employees from 40 hours to 37.5 hours per week; changing the Success-Share (the employee incentive program) so that bonuses are based on overall worker production rather than individual performance; cutting down on overtime; emphasizing the importance of cross-training instead of the previous "one job for life" philosophy maintained by the previous owners. Added to all of the above changes is the fact that the new owners have designated 1995 as the year when pay bands (scales) will change depending on how many jobs the worker has learned and the level of difficulty of his jobs.

One of the many positive things that the new owners have done is the implementation of the Basic Skills classes. Math, reading, writing and ESL classes are offered, some of them on all three shifts during a worker's lunch or dinner break. The company pays .5 hour, and the employee gives .5 hour. GED classes have also been offered on the employees' own time.

My class has five students at the present: our class is on the second shift and meets from 5-6 p.m. three times a week. I started out with 10 students on the roll, but due to shift or plant changes, of which the worker had no control, I am down to five students. The class is all male and ranges in age from late 20 to near retirement. All are English-speaking and have a high school diploma; most are long-term employees of the company.

The math classes have began three years ago with a very general curriculum. However, during the course of previous sessions and my projects, the curriculum changed somewhat, as the coordinator tried to implement more job-related examples with which to teach various math concepts. I was asked to use materials other than the ones with which we began. Unfortunately, in some cases, there was not the time to adequately get precise examples or proofread the ones we did have. Therefore, sometimes mistakes were caught by the students, who are under the impression that the teacher should know everything. I give this background information to let the reader know the various outside influences that impacted my original question.

METHOD

Three things that I did during the course of my project helped to make my question somewhat clearer to me. These were:

✔ Conducting ongoing interviews with the students in my class as well as other workers to see how much metric measurement was actually used;

✔ Interviewing supervisors of my students plus other plant supervisors to find out their impression of the metric system used in the plant; and

✔ Using another curriculum, along with the generic one, to get more precise metric measurement.

I wanted to see what my class knew about the word "metric" so we brainstormed a few ideas. I asked them to think of any word or phrase that came to mind when they thought of metrics. Some of their ideas were "foreign, groceries, millimeter, grams, rulers, hard, soda, measurement, charts, measuring tool." This proved to me that they had some familiarity with the metric system. I then gave each of them a ruler with centimeters and millimeters one side and inches on the other. We set out measuring everything we could: chalk, erasers, pencils, books, paper clips, blackboard, wheel diameter, thickness and the size using all three of the measurements. We then looked at 2-liter soda bottles, soda cans with milliliters on them and milk cartons. In this way they could get the idea of comparing metric units with English units. They could see how small some and how large others were. Later we would go over some of the measuring tools that they use at the company.

We briefly reviewed fractions and decimals and how to change smaller and larger units by either multiplying or dividing. Since our own number system is based on powers of ten, the metric system should be easy to understand. Learning new ideas about number relationships will hopefully provide a foundation or pattern for understanding number estimation, and will lead to a better understanding about the connections between decimals and the metric system. The Mathematical Connections standard seems to permeate so much of our math thinking, and I feel it makes math seem more relevant to one's everyday like. Reading from this standard, "When mathematical ideas are connected to everyday experiences and life skills, learners become aware...of a useful tool applicable to the real world."

I used a very general metrics curriculum that was put together by a fellow teacher. It is very basic and goes over the conversions that will be helpful to the workplace as well as those that we come across everyday. I felt this would be the appropriate curriculum to begin with in this class and then expand on it. The conversion units used by this teacher were a little different than the ones used in the factory. Many times this curriculum's conversions were not as exact or had rounded off answers, and for one of my students, this was very confusing, because he needed calculations to the ten-thousandths place. What I had suggested on several occasions was that students would work the problems in this unit, and do the answers to the decimal unit that would be most useful to them at work. I encouraged them to work on estimation skills, and when a mistake was found in the curriculum, I suggested problem-solving techniques such as working backwards to get the answer or eliminating other possibilities as answers or using similar or simpler problems to come up with the answer. However, the students easily became frustrated when their answers did not match the ones in this unit. I found another metric conversion chart that was more precise than our first one, but even when the answers were slightly different, the students became easily confused.

I suggested that they bring in samples of their own work where metrics would be applicable. One student, who had insisted that he used absolutely no math in his job, brought in examples of the blotters that he puts on wheels. Some of the blotters measurements were given in metric units. This worker had a chart in front of him to which he referred. However, I wanted him to see the connection between the metric and standard units, so if need be, he could do the conversions with paper and pencil. Another student needed help with wheel tolerances (measurement differences that are allowed without affecting wheel dimensions) showing them as decimals, fractions and metrics. Probably the most eye-opening situation for me was when one of the students said that he never realized that 3.25 and 3 1/4 were the same. He had looked at a chart of hole sizes for years and always thought that 3.265 and 3 1/4 were the same, because that is the approximate conversion that his particular chart showed.

I found an easy way to teach how to change the metrics conversion by keeping two words in mind: DOWNRIGHT and UPLEFTING. I gave the students what looks like a vertical number line *(see Appendix A)*. The units are in the middle and above and below are the metrics prefixes. Depending on what you are looking for tells you to go either up or down. For example, if I want to convert 5 decimeters to kilometers, I look at my number line, and see that kilo is up 4 spaces from the deci. I know going up means moving the decimal to the left, in this case 4 places to the left. 5 is the whole number which means the decimal point is right after this number, so I would move the decimal 4 places to the left, which means adding 3 zeros and I end up with .0005 kilometers.

The millimeter is the main metric unit used, and it shows the length or width of a wheel and its diameter as well as its hole size. One of the students work closely with the wheel's hole size; in talking with him, he showed me the hole size gauges that are in metrics as well as standard. He knew from the charts that 203 mm is the same as 8 inches, 254 mm is 10 inches, etc. Depending on whether orders come in using the metrics or standards, he uses the appropriate gauge. Accuracy is very important here, because the holes must be very tight, within 0.0025 in. He went on to say that summer and its heat bring on new problems, because the heat makes the hole tighten on its own and it may not be the true fit.

The metric measurements that I stressed are grams, liters and millimeters. The weight of the wheel may be given in grams on the work check, in which case the worker needs to convert that to pounds; or the mix or bond may be given in grams, ounces or pounds and must be converted appropriately.

I spoke with different people in the plant about measurement tools. A tool called the micrometer has fine discrimination and high reliability. "Mikes" are reasonably safe instruments to use for measuring no finer than one-thousandth of an inch (0.001 in.) discrimination and can go up to 5 in. (127 mm). Metric micrometers are scaled to measure up to 0.01 mm. For measuring tighter tolerances up to 0.0001 in. or 0.002 mm, the vernier mike can be used. At very close tolerances (anything over 0.0005 mm.), the care you will have to use with a vernier micrometer will usually take a lot of time. Students brought in these different measuring tools and we helped each other with measuring a wheel, its diameter, hole size thickness. One supervisor told me that average tolerances for a wheel are usually between plus and minus 0.010 and 0.020 in. If a customer wants a tighter tolerance such as 0.004 in., the rejection rate of the wheel goes up. A supervisor from another plant told me that, in some cases, he has seen metric measurement tools actually save the company money, because the mm tolerances, especially if they are in whole numbers, are measured to a wider tolerance than decimal-inch counterpart. This supervisor gave me an example that by going from inches to mm, he has seen tolerance limits become broader, and for the company, that means less finishing is needed for the wheel. He went on to say that the limits might be plus 0.015 in., minus 0.000 in. If, he said, the limits were plus 1 mm, minus 1 mm, then the limits are more than double and this broader range could be cost saving for the company.

FINDINGS

One positive change that I feel happened with this project is that some of the students did find that metrics were useful to some of their jobs. A student said that now he could do the appropriate calculations whether in metrics or standard.

In writing their evaluations of the class, one theme came through and that was that many of the metrics projects were helpful now and would be helpful on other jobs if the workers were cross-trained in the future. Even though their pessimism came through, I am hopeful that these workers will gradually see their thinking change as the American job market changes.

In interviewing one of my students who was the most vocal on the inaccuracies in the curriculum materials, (his job is very precise), he said that he is slowly being trained on a new very expensive machine called the OMNI. This machine takes the wheel, after the kiln process, and grades it, sides it, internal grinds and outer grinds the wheel's diameter. He sees his future job as much less hands-on and more as monitoring the programs for this and other machines like it.

By 1995 or later, the work force should definitely be different less labor intensive. Hopefully, workers will be able to do many varied jobs and when a worker is on vacation, there could be several people who could do his job. The Mathematical Connections standard (Massachusetts Adult Basic Education Math Standards) summarized this when it said, "The learner's ability to make mathematical connections make the learner a more flexible, efficient, and productive worker."

In summary, an overview of my findings are:

- ✔ supervisors and students see precision as necessary in the workplace and must be taught according to their standards, using **their** job-specific tools.
- ✔ I discovered that metrics are used approximately 25% of the time, whereas standard measurement is used the other 75%. Metrics are not as all encompassing as I first thought. Metrics do give a worker another form of measurement in the workplace.
- ✔ many other factors affect a worker's performance; learning math can be taught as a connection to work and life skills, but other factors must be taken into consideration.
- ✔ crosstraining must be incorporated into the workplace, so that a worker can see the process of the total product, not just one aspect of it. Teaching math is much the same: concepts must be taught in context, rather than just one at a time.

What these experiences showed me was what I had read in one of our math standards that adults seem to be very concrete learners and want all numbers to come out exactly, just like the answer sheets. The Standard: Math as Communication said..."adults too easily become obsessed with getting the (only) right answer. This limited exchange between the learner and the answers in the back of the book makes math learning an isolated experiences that is clearly at odds with the ways in which math is used everyday."

CONCLUSION

Making connections with the real world helps students to see the usefulness of mathematics and understand that the metrics can be another way to better solve some workplace measurement problems, but not the only way... just one more method. When I presented real work problems in various contexts, understanding was more apparent. However, as I tried to make metric activities show a relationship to their environment, I do not feel that this goal was completely attained. I read in an *Arithmetic Teacher* that, "limited mind sets and understandings stem for students' having limited experience in mathematics." This article went on to say that "over time as mathematical connections are made through various enriching experiences, students can view mathematics in a way that allows them to glimpse the whole picture rather than just a part. That mathematics is a relevant part of other activities, and that mathematical thinking is useful for solving many problems in and out of school." I do feel that I began this process where these students might think a little differently about metrics, and hopefully in future classes and the workplace, this concept will be expanded. I feel this could be an implication for further research: to explore a curriculum that uses math as it takes the wheel from its very beginning to its conclusion. Cross-training may be the beginning of this research; math class could carry it on.

From this project, I learned to reevaluate my own thinking on the basis of new information and recognize such reappraisal is a normal process that each of us should continually work on. I know for myself I saw math in such a different light, in relationship to the Standards. This project really made me look at the three standards that I was concentrating on. It made me realize that math does not stop in the workplace or the classroom, but continues everywhere around us. The Standard on Mathematical Connections said it best:

"When mathematical ideas are connected to every day experiences and like skills, learners become aware of the useful of mathematics beyond the classroom, and of the importance of math in all career choices..."

BIBLIOGRAPHY

Henry, Boyd. *Teaching the Metric System*, Weber Costello, Chicago, 1975

Rasmussen, Steven and Spreck Rosekrans. *Key to Decimals*, Books 1-4, Key Curriculum Project, Berkeley, CA., 1985

Gill, Alice. *Arithmetic Teacher*, "Multiple Strategies", pgs. 380-386, vol. 40, no. 7 March, 1993.

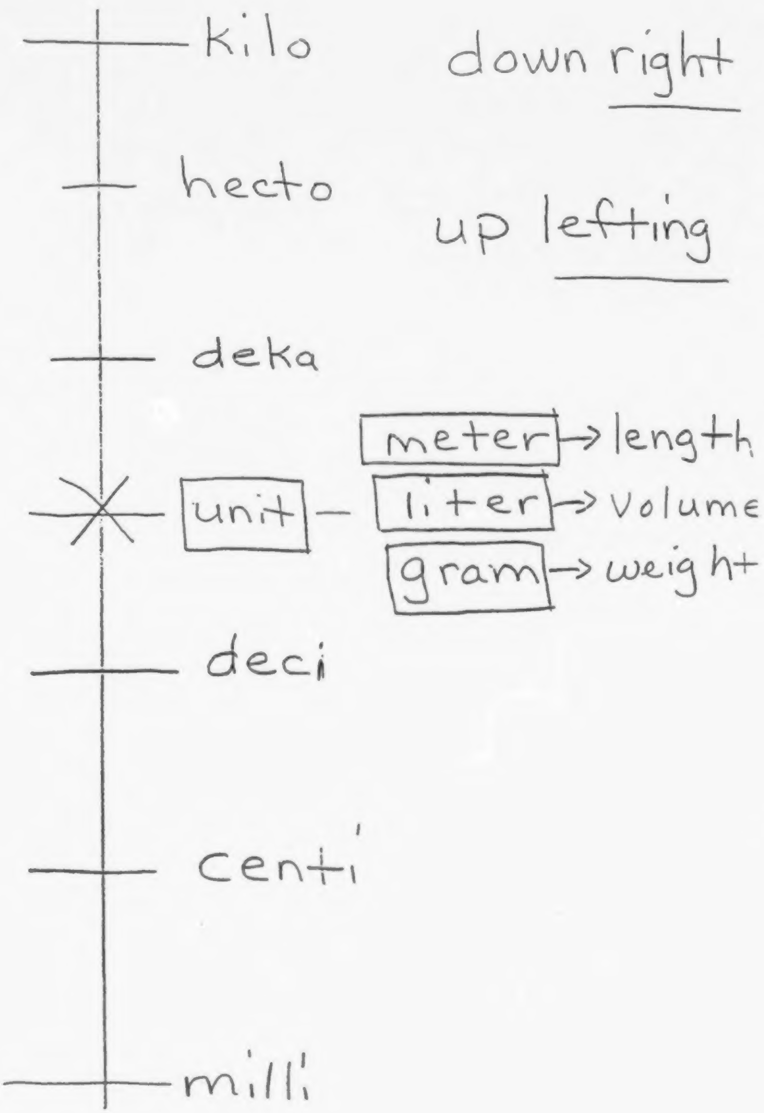
Arithmetic Teacher, "Empowering Students Through Connections", pgs. 289-299, vol. 40, no. 6, February, 1993.

Kempf, Albert and Thomas J. Richards, *Exploring the Metric System*, Laidlaw Brothers, Publishers, River Forest, IL., 1973.

Odom, Jeffrey V. *Successful Experiences in Teaching Metrics*, U.S. Department of Commerce, issued January, 1976

Henry, R. Lee, *Systems 3 Science*, Follett Publishing Company, Chicago, 1975.

APPENDIX A


kilo
down right
hecto
up lefting
deka
meter → length
unit
liter → volume
gram → weight
deci
centi
milli

MATHEMATICS AS PROBLEM SOLVING

NON-TRADITIONAL PROBLEMS IN THE FOREFRONT

By Marilyn Moses, Adult Learning Center, Brockton

THE STORY OF THE QUESTION

What would happen if part of each class (35 to 40 minutes) was spent solving non-traditional problems (games, open-ended problems, reasoning problems) without the use of paper and pencil? Learners may use manipulatives, calculators or mental math and see if this technique enables the learner to understand mathematics in their life beyond computation, to become better problem solvers and to become aware of numbers in their lives and and how the numbers affect them.

Many adult learners seem to be coming to math class terrified of having to do math. They arrive with preconceived notions that doing math means completing a page of computation problems appropriate for their level whether that is dividing whole numbers or working with decimals, fractions, or percents. Many don't make any connection to "doing math" and how they all must use numbers in their lives.

When I was a junior high school math teacher, I was driven by the text. I was given very specific guidelines by the math department head of what material needed to be covered in the following year. The most important thing was that each teacher keep nice, quiet control of the classroom, have students memorize the formulas (don't think or question), follow the "pace and emphasis chart," and make sure that the students were ready for the final exam. This exam was not only to record the students' progress but was also an indicator of how well the teacher had completed the program. This didn't leave very much extra time for "math games" or "brainteasers." Those types of fun math exercises were saved for days before vacations or filler activities. This is the baggage brought with me to adult education. Never before did I have permission to bring these types of activities to the forefront where I've always suspected they belong.

> ***"A lifetime of negative experiences or memories of the education process has produced in many (adult) students a major lack of self-esteem and self-confidence. This lack prevents the individual from attempting important risk-taking tasks involved in thinking."***
>
> MASSACHUSETTS ADULT BASIC EDUCATION MATH STANDARDS, STANDARD 1, MATHEMATICS AS PROBLEM SOLVING

In one class, I asked a group of GED math students to tell me how much it would cost if I bought 4 shirts for $7.98 each. They were told they could figure it out any way they wanted, except they could not use paper and pencil. I watched as they used their fingers in the air or "wrote" on the desk. Most were able to multiply and get the right answer. When I asked HOW they got their answer, all agreed that they needed to multiply $7.98 by 4. I then asked if they were in a store and had to figure out the same problem would they have done it the same way. All agreed they probably would not solve it the same way in real life. Some said they would have multiplied 4 by 7 plus 4 by 1 and then subtracted eight cents from the total. Others said they would have rounded $7.98 to $8.00, multiplied that by 4 and then subtracted eight cents from the product. I then asked why no one admitted to solving the problem like that in class. The response was that this was a math class so they needed to calculate it out.

It concerned me that they didn't value their own intuitive method of solving a problem. Only if they "did it out" did they see it as doing mathematics. There seemed to be a direct correlation between their math feelings and my former math teaching.

These issues seemed to be addressed in the K-4 Standard Six of *NCTM's Curriculum and Evaluation Standards for School Mathematics* "Number Sense and Numeration" where it states, "Intuition about number relationships helps children make judgments about the reasonableness of computational results and of proposed solutions to numerical problems. Such intuition requires good number sense." It seemed to me that my students' number sense needed to be developed and appreciated. This is also presented in Standard Six (for grades 5-8) "Number Systems and Number Theory," that developing number sense "improves problem-solving capability by providing a better perspective of arithmetic operations."

Standard One: Mathematics as Problem Solving states, "Problem solving is the process by which students experience the power and usefulness of mathematics in the world around them." Our learners need to view mathematics as a thinking process and not just computing and memorizing formulas.

CONTEXT

My math class meets every Friday from 9 a.m. to noon at the Adult Learning Center in Brockton, Massachusetts. This is a pre-GED class and every student in the class has elected to take this class. It is comprised of fifteen students who are all in other reading classes which meet twice a week for three hours each. There are three men and twelve women. Six learners identify English as their first language; four named Portuguese as their native language. Creole is spoken by three of the students; one speaks French; and one identified Hmong as his first language. One woman is under 24 years of age and the fourteen other students are between the ages of 25 and 44. The reading scores from TABE (Test for Adult Basic Education) ranged from a low of 3.5 to a high of 8.4 and an average (mean) score of 5.2. Students should be able to add, subtract, multiply, and divide whole numbers to be placed in this class. The class meets only once a week, and because many students need to miss an occasional class due to work schedules, babysitting problems, sick children, their own illness, in-service release days, transportation problems, or doctor's appointments, there very often is loss of continuity in the lessons; therefore, lessons need to be self-contained.

All but one of the students have identified a GED as their goal. One of the students has her high school diploma and attends the learning center to improve her reading and math skills.

Below, I describe two of my learners more in depth. I picked these two because I saw the greatest change in them. When I describe what we did in class, I include their responses.

Denise is a very self-confident, white, English-speaking woman with an outgoing, magnetic personality. She is 28 years old and has two little girls. I vaguely remember Denise when she was in junior high. She had no use for school and was a real party girl. Denise dropped out of school in the eighth grade. She has been a student at the Adult Learning Center off and on for almost ten years but had never taken a math class here.

This year she made the commitment to follow through with her education and expressed that she viewed herself as a "math phobic." I was warned by the counselor to "handle with care" as Denise had no self-confidence in her math abilities. Although her reading score was 7.4 on the TABE, she entered my math class not knowing her multiplication tables and not being able to divide using the algorithm. Yet, she was determined to overcome her fears and give it her best effort. Denise also worked with a tutor once a week for one hour. From Denise's math survey, she described math as "something we all have to do. I don't think it is something we really have to like, but you have to keep an open mind." Denise also confided in me that she has a learning disability which she says makes learning math very difficult.

From Denise's math survey, she described math as "something we all have to do. I don't think it is something we really have to like, but you have to keep an open mind."

Laura is another woman in the math class. Like Denise, she too has been at the Center for many years off and on. Her TABE score in reading is 6.2, and she also dropped out of school in the eighth grade. Laura has a very negative attitude towards math, specifically, and life in general. She was not comfortable participating in class but was very loud, disruptive, and silly. Often when she worked in workbooks for homework she would skip pages or problems that were difficult for her and make excuses why she didn't do it. Usually she was late for class. There was always an excuse — doctor's appointments, church meetings, distractions from her children or family, shopping, missed the bus, etc. Nothing seemed to interest her and most of the time she would miss the point of a discussion and go off on a tangent. Whenever I gave her an assignment, her usual response was "I hate to do that." This was her way of not having to admit that math didn't make much sense to her. She didn't put much effort into her work and was always very content to have the teacher or another student hand her the answers. I suspect Laura's connection to the learning center was more a social outlet than any great desire to achieve in math. Laura responded that math makes her feel "like my head is going nuts."

Laura responded that math makes her feel "like my head is going nuts."

METHOD

I started this project by explaining to the class what the project was about and had each student fill out statistical information and a survey pertaining to their feelings about mathematics. On that survey, I asked them to complete the following:

1. MATH IS...
2. IF I WERE A MATH TEACHER, I WOULD...
3. I LIKED MATH UNTIL...
4. WORD PROBLEMS ARE...
5. MATH MAKES ME FEEL...

At the end of each class, students filled out Learner Logs that asked the following questions:

WHAT DID WE DO TODAY?

WHAT DID YOU THINK OF IT?

WHAT DID YOU LEARN?

For the next eight weeks of class, I had a rough idea of the type of problems I wanted to use, but collected the problems as I went along based on what happened the previous week and what seemed to be the needs of the learners. I tried to find activities that would engage all students and allow them to explore and solve problems.

I began with cooperative logic problems, to get the students to feel comfortable working together, then on to a newspaper activity to see the importance of numbers to the meaning of a text. This led to an activity where the learner had to fill in the numbers in a story problem so that it was meaningful. They then had questions to which they had to give reasonable estimates. At this point, it was becoming clear to me that my learners were having difficulty dealing with and making sense of large numbers, so that the last three lessons focused on working with large numbers.

In each of the activities, I tried to stay out of the picture as much as possible. I was interested to see if the learners would be able to trust another student and/or their own math ability to reason, solve, and justify their own solution. Would they think about the reasonableness of a solution and not rely only on rote learning and memorization of algorithms and formulas? Also, would they be able to validate their way of thinking and to understand that mathematics is thinking and not just computing?

FINDINGS

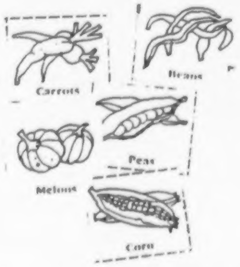
The project began with each learner describing what they thought math is through a math survey. Most considered math to be important and interesting but hard to do and mind boggling. Almost all expressed that the teacher's attitude towards her students and subject the most important in helping them learn math. Their surveys indicated many of them had had poor experiences in the past with math classes. All expressed a lack of understanding as a reason for their dislike of math and most identified fractions and decimals as their falling off point. All students identified problem solving (word problems) as "difficult," "hard," "hate them," and thought of them as "the enemy." When asked how math made them feel, words like "headache," "going nuts," "uptight," "intimidated," "nervous," and even "shameful" were the responses.

Week One. The first class was a non-computational cooperative logic problem called Plant-A-Garden. This was the first time any of the students worked in small groups. They seemed a little hesitant but wanted to cooperate with me. Five groups, each with three students, were formed randomly. They were given an envelope with clues in it which must be read aloud but not shown to another member of the group. This activity forced the learners to talk to each other and to listen to each other.

In the small cooperative group, Denise was the leader. Her group was the first to "finish" the problem, and she called me over to verify their solution. They were very proud of themselves until I suggested they read their clues over again and check their solution. They were very surprised the clues didn't fit their solution. Another student in the group then admitted she didn't think they had solved the problem correctly. As the group continued to work on the problem, their frustra-

Plant A Garden

Sample Clues



These are your clues to help solve the group's problem. Read them to the group, but do not show them to anyone.

Problem: Which crop is planted in each of the sections of the garden?

- The beans are planted in front of the corn.
- The peas are next to the tomatoes.

— — — — — — — —

These are your clues to help solve the group's problem. Read them to the group, but do not show them to anyone.

Problem: Which crop is planted in each of the sections of the garden?

- The peas are next to the corn.
- The melons are to the right of the carrots.

Fraser, Sherry, *SPACES: Solving Problems of Access to Careers in Engineering and Science*, Dale Seymour Publications 1982, p. 63. © 1982 The Regents of the University of California tion levels rose but no one gave up. The group persevered and correctly solved the problem, much to their delight and satisfaction.

One thing that surprised me was the difficulties that many of the students had with the language. Terms like "in front of," "next to," and "to the right of" gave many of them trouble. I maintained my role as questioner by asking them "Do you all agree?" "Is that what it means?" "Are you sure?" This led to a lot of discussion in the groups and finally agreement.

One of the five groups didn't seem to work out so well. One woman, who has been with me for years, completely took over the group. She told the other members when to read their clue, she moved the pieces of the puzzle without consideration for the others and completely dominated the group. When I focused on the group and explained that they all had an obligation to take part in the process, the other members of the group seemed overwhelmed and continued to back off. This group finally solved the problem when a student from another group came to the rescue. All three students in this group expressed frustration in working with each other. I realized I needed to address the issue of group dynamics in the future. Also, I was pleased to see the positive response to the other student's intervention.

Learners responded that this type of problem made it clear to them the important of reading and listening carefully to understand what the problem is asking. I observed as the frustration and confusion of solving this problem turned into delight and pleasure as each group solved their problem. Most students expressed that they enjoyed working in groups.

Two things struck me: one, some were very quick to look to me to verify their solution and two, others were sure they had solutions when in fact no attempt had been made to verify their work.

Week Two. This week we tried other cooperative logic puzzles, Mystery Number and Secret Number, but these problems involved numbers. This time when I broke the class into groups, I made sure my problem

What is the Mystery Number?

The number is even and less than 30.

8 is not one of its digits.

The number is neither 20 nor 24.

The sum of its digits is even.

One of its digits is 2 (but the number is not 22).

The number is more than 10.

The number is not 22.

The number is less than 30.

Goddard, Ruth, et al, ***Strength in Numbers: A Resource Book for Teaching Adult Numeracy***, Victoria, Australia 1991, pp. 18-19. See also *Get It Together* by Tom Erikson, Equals Project, Lawrence Hall of Science, Berkeley, CA 1989, for other problems and suggestions for making new problems.

group was separated. I gave them the envelope of clues to solve the problems, and students worked on their problem but again looked to me to tell them if they had solved it correctly. This week they were beginning to realize they would have to read the problem more than once to solve it and verify their solution. Many of the students needed an explanation of some of the math terms that were in the problems, such as "digit," "odd," "sum," and "difference." One of the groups needed a jump start. They were becoming very frustrated and had no idea how to proceed. I suggested using a process of elimination to get them started. They were off and running.

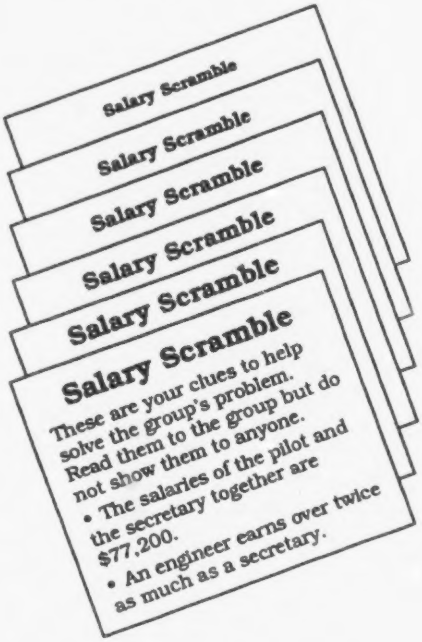
The next problem was Salary Scramble and this involved very large numbers. Most found this very challenging. Some learners opted to use calculators to assist in solving this problem and only one group was able to come up with the solution in the class. I heard Denise comment that "A teacher couldn't make more money than a pilot. We must not have this right." Some students took it home to work on and share with their families. Denise took hers home and returned the next week with the problem completed. She said her husband helped her with it. Another student said she tried one of the games at home with her first grader and was going to try to make up some easier puzzles for him.

From the logs I sensed students had gained an appreciation of reading the problems carefully and were relying a little less on me to tell them how to solve the problem. They seemed to be devising some strategies of their own to approach the problem. Denise said she learned "to ask questions, a lot of them." "That I have to read very carefully, that is very good for the mind," commented Lois. Doug wrote that he learned to "read and understand math language."

I was amazed at how well most students took to working together. They left the room smiling and looked as if they had had a real workout!

Week Three. The Newspaper. The learners each had a page from the newspaper and a questionnaire to

Salary Scramble



Salary Scramble

These are your clues to help solve the group's problem. Read them to the group but do not show them to anyone.

- The salaries of the pilot and the secretary together are $77,200.
- An engineer earns over twice as much as a secretary.

Fraser, Sherry, *SPACES: Solving Problems of Access to Careers in Engineering and Science*, p. 69.

fill out pertaining to their page. They had to look for a number on a specific page of the newspaper and see how that number was used, was it an estimate or an exact number, and how was it meaningful to the sentence. After each learner had completed their page, they took turns sharing their results. We made a list on the board of all the different ways numbers were used in the newspaper. Also, we discussed if the number in the paper was an exact number or it if was used as an estimate or rounded number. The learners then took a marker and underlined every number on their page. Many were surprised to see how many numbers were actually used and all the different uses. Only two students could find an article that did not have a number in it!

Learners seemed to have trouble reading and comprehending large numbers and also determining the difference between an exact number and an estimate. From the logs, many of the students commented that they now realized how important numbers are in their lives. Laura said, "It made me see how much numbers are used in everything we read or do." "I never look at a newspaper the way you shown, how many numbers are there for many different reasons," wrote Lois.

Week Four. Thinker Math. On the overhead, I displayed the following demonstration story):

#1. Eric and Jan played math music trivia. In the first round, Eric answered questions about a piano. He said, "There are_______keys on a piano. There are _____ more white keys than black keys. So, there are________white keys and ______ black keys." Jan answered questions about the composer Beethoven. She said "Beethoven wrote ____ symphonies and _____ piano concertos. The number of symphonies is one more than twice the number of piano concertos."

16 9 52 36 4 88

Thinker Math: Developing Number Senses and Arithmetic Skills (grades 7-8), by Linda Schulman, Carole Greenes, and Rika Spungin, 1989 Creative Publications, page vii.

Newspaper Activity

Each student has a section of newspaper.

1. Find a number on one page of the newspaper and write it down.
2. Write the headline or sentence that contains the number.
3. Rewrite the headline or sentence, leaving out the number.
4. What meaning does the number have? How does omitting the number change the meaning of the headline or sentence?
5. Is the number exact or an estimate? How can you tell?
6. Does a newspaper article always have at least one number in it? Can you find an article that does not have a number in it?
7. Show why numbers are important to a newspaper by giving five ways in which numbers are used in the newspaper.

"From the File" of Bonnie V. Whitley, *Arithmetic Teacher*, Vol. 40, No. 5, Jan. 1993, p. 290.

Their mission was to fill in the numbers so the story made sense. The students were very quick to identify the relationships between the numbers. We were all happy to learn that there are 88 keys on the piano, a fact nobody in the class knew. From the context, they realized that the sum of two of the numbers had to be 88. They also interpreted the phrase "one more than twice a number." No problem!

The next story was a little more difficult because it involved decimal numbers.

The fastest pitcher on record is Nolan Ryan. On August ___, ____, he pitched a ball at a speed of ___ miles per hour. The ball took less than ___ of a second to travel the distance of ___ feet ___ inches from the pitcher's mound to home plate.

0.5 ***60*** ***100.9*** ***1974*** ***20*** ***6***

Thinker Math: Developing Number Senses and Arithmetic Skills (grades 7-8), by Linda Schulman, Carole Greenes, and Rika Spungin, 1989 Creative Publications, page vii.

Students recognized 1974 had to be a year. Some also realized that in the story, the reference to inches was probably a number less than 12. Others were quick to realize that 0.5 went on the blank "of a second" because it was the only number less than 1. That left 20 as the only possible choice for the date. The two remaining numbers led to a discussion of the distance from the pitcher's mound to home plate and the miles per hour the ball travels. Finally, the class agreed that the distance must be 60 feet and the ball traveled 100.9 miles per hour. As a class, we tried two more demo stories and then learners were asked to solve another page with the option of working alone, in pairs, or small groups. Some immediately went to the group they had worked in previously, particularly if they had been successful together. Others worked alone but when they were stuck, looked to another student for support (not me), and groups looked to other groups for support and eventually the class became one very large group. I insisted that they not change any of their answers until they were truly convinced a change was needed.

This class was taped and because of all the discussions and laughter, sounded more like a party than a math class. I've rarely had a math class where people laughed so much. According to the logs, many students commented on the need for reading the problem more carefully and looking for context clues (reasonableness). Most seemed to really enjoy this activity and one student said, "This paper makes word problems seem a little easier." Laura commented that "numbers are important to what you read." Another student commented on the relationship between the numbers. Some students asked for more to take home to their families!

Laura liked this type of exercise very much. She had not had much success up until this point. This exercise was not as threatening to her because "all the answers were there." In her log, Laura wrote that "it also helped you learn to read the problem better." She learned "that the right numbers are important to what you read. It also got me to add, multiply, and subtract. I also learned greater and less."

It was becoming clear that language was much more of a problem than I had realized and that most students had never thought about the numbers in their lives.

Week Five. Quick Questions.

You wish to dispose of $1 million. So you give it away $50 every hour. About how long will it take you to give it all away?

Approximately how many seconds are there in a day?

Counting labels, signs and everything else, how many commercial messages does the average American see in one day?

About how many times a day does the average American blink?

In 1983, over 45% of automobiles sold in the United States were purchased by women. What do you estimate to be the percent of microcomputers purchased by women in 1983?

About how many times a day does the average American laugh?

How many miles long is the line drawn by the average pencil?

How many 12-ounce cans or bottles of soda does the average American drink in a year?

If 600 bottles are to be put in cases that hold 24 bottles, about how many cases will be needed?

Jean Kerr Stenmark, Virginia Thompson, Ruth Cossey, *Family Math*, Lawrence Hall of Science, Berkeley, CA 1986, p. 231.

Most of these questions are not what you might think about everyday. The students' responses had to be off the top of their heads. No paper and pencil. No calculators. This was to be an exercise in estimation. Most of the estimates were way off base. Any questions that required thinking about large numbers was way off. For example: On the first question which refers to disposing of $1,000,000, the estimates ranged from 5 hours to 300,000 (no units). I found that the question needed to be clarified to supply a unit of measure the answer should be in and that most of the students had no concept of one million.

Only when the students had a frame of reference to fall back on did the response come close as in the question of "How many bottles of soda does the average American drink?" Most students took their own consumption into consideration and formed their estimate based on their own drinking habits. One student estimated that an average person might drink one can of soda a day; therefore, guessing 365 cans of soda was very close to the given estimate of 359. Others guessed up to 4 or 5 cans of soda a day and, therefore, they overestimated. We took a poll in the room and found that most students in our class drank under the given estimate. It seemed this was a question that most students could relate to and, therefore, were able to make more meaningful guesses.

Clearly, many of these learners need exercise that will develop their number sense in estimation and numeracy. This may make a good diagnostic tool in assessing a student's skills in estimation.

A Look at 1,000,000

Suppose you tried to count to 1,000,000 and counted just one number each second, without taking any time out. How long would it take? (Calculators will be useful.)

What about a million days? How many years would that be? Will you be alive a million days from now?

How many days old are you now? How many hours old are you?

How much money would you have if you had 1,000,000 pennies?

How much money would you have if you had 1,000,000 nickels?

How much money would you have if you had 1,000,000 dimes?

Which pile of a million would be bigger?

Week Six. Look at A Million. As a result of last week's estimating, I realized that my students seem very uncomfortable with large numbers so we looked at a million. In this exercise, they were asked to calculate how long 1,000,000 seconds was. Many found this very confusing. Most were able to figure that there were 60 x 60 x 24 seconds in a day, although they weren't real sure what to do with that number. Next, they calculated how many years one million days is and were very surprised to find out that this was more than 2,739 years. Even that number didn't seem to make an impact until I mentioned that the year was 1993 and 2739 years ago was 746 years before Christ! Then each student calculated approximately how many days old they were and seemed surprised at how young we all were in relation to 1,000,000.

Laura's interest was sparked when we discovered we were very close to the same number of days old. This was something she could relate to. The next week when Laura came to class, she said everyone in her family had figured out how old they were in days. I was surprised at how unaware my learners were in working with large numbers and that I don't think they had ever considered what a million is.

This activity also posed the problem of what to do when dividing and working with remainders using a calculator. Many students seemed to know what operation needed to be applied, but had no idea how to express their answer. For example, when 1,000,000 was divided by 86,4000 to find the number of days in a million seconds, the result was 11.574074. No one had any idea what that decimal stuff represented.

Week Seven - Worldwide News.

Suppose a baby was born at midnight, and you told one person about the baby. Then that person told two others within 10 minutes. And then those

two each told two others in 10 minutes. And this continued all night long, with every person telling two others every 10 minutes who hadn't yet been told. How many people would know by 8 o'clock in the morning?

Marilyn Burns, Math for Smarty Pants, Boston: Little Brown & Co., 1982, p. 123.

Students decided to work in groups and to use calculators to solve this problem. All students approached the problem willingly and decided that they needed to find how many 10-minute intervals the problem was referring to. Then, they saw the problem as just multiplying that number by 2. They were going nowhere fast with this problem and were all sure they had the correct answer. I then decided to start them off by diagraming what this meant.

I

10 min. I I

10 min. I I I I

10 min. I I I I I I I I

10 min. I I I I I I I I I I I I I I I I

and so on...

This seemed to get people on track but they were overwhelmed when the calculators maxed out. They were amazed at how large the numbers go so quickly. We discussed how rumors might spread and also the relationship this has to some of the pyramid type chain letters. I was pleased when Denise recognized this as very similar to a shampoo advertisement where a girl tells two friends and they tell two friends and so on.

I don't think many students got much out of this exercise. Comments were "overwhelming" and "confusing." None of the students had ever looked at exponential growth before and it didn't make much sense to them.

If I were to use this exercise again, it would need a stronger introduction. This would have been a good place to bring in pennies and demonstrate what would happen if you start with one cent and keep doubling the amount you have.

Write A Statement

Write a true statement using each of the numbers.

1. Use a number from 1 to 70 in a true statement.
2. Use a number larger than 100 but less than 500.
3. Use a number between 1,000 and 2,000.
4. Use a number larger than 20,000 and less than 100,000.
5. Pick a number larger than 500,000 and less than 1,000,000.
6. Use a number that is larger than one million and less than two million.
7. Write a statement using a number greater than one billion.
8. Write a statement using a number less than 1.
9. Write a statement using the largest number you can imagine.
10. Write a statement using the smalles number you can imagine.

Week Eight. The End. Still concerned that numbers just don't have a lot of relevance to them, I asked them to write a statement using a specific range of a number. Most students did very well and seemed to be accurate on the first three questions. These were numbers that they were familiar with. As the numbers became larger *(Questions 4-7)*, the statements became more vague and less exact. Two students referred to numbers that had been in previous activities. Statements usually referred to money and were estimates or rounded values instead of exact numbers.

When they were asked to think of a number less than 1, some of the students used 1/2 or 1/8 or .5. Others thought 0 was the only number smaller than 1.

Although they were only five of the original fifteen students, I asked them to fill out an ending survey form. Laura stated that the project "helped me to learn how important numbers are in everyday life." When asked her impression of the class, she wrote "A piece of cake. Math is not so bad to learn and not to say you hate it. It made me feel that math can be fun if you look at it as a game." This was quite a change in attitude for Laura. She seems more willing to try to work on a problem, and I heard few complaints from her.

They realized the importance of numbers in their lives and felt they had learned the importance of estimating. One student said she wishes there had been more percents, fractions, and algebra, but also stated that the types of problems we worked on forced her to read carefully and to think more about the problem.

The 35-40 minutes I intended to spend on the project became 1 hour to 1-1/2 hours. Students seemed truly involved. Often learners asked to take extra material home with them to work on with family members. On the last day of the class, I asked what we had done in class this term. I asked what the first activity was and was astounded that they could answer "Plant-A-Garden." I then asked what was the next activity and again received the response of "Salary Scramble." We proceeded to list the activities on the board. They had remembered not only what we had done but in the order we had done them. I don't think that if we had followed a textbook the response would have been the same!

CONCLUSION

I saw growing confidence in the ability to attack a problem in some students. I saw comfort levels expand and an appreciation for the importance of numbers in their lives. They were relying less on me and more on themselves.

I think the most important component of a class, especially for adults who have probably had difficult experiences with math in the past, is to provide a non-threatening and supportive environment where the learner's thought process is valued, and they are free to explore ways to solve problems.

It became clear to me that language and math vocabulary were much more troublesome than I realized. I felt they got a lot out of the *Thinker Math*, and I see this not only as a an exercise in estimation and reasonableness, but a great way to get students to read more carefully and truly understand what the words mean. This would make a great introduction to word problems, especially with students whose first language is not English.

Through this project, I discovered that any number, particularly a large number, that doesn't have any relevance to their lives becomes meaningless.

What surprised me the most was how receptive the learners were to try something different. Because we all had so much fun, many were anxious to share what they had done with other members of their families. Sometimes this was to try activities with young children or to share it with older children or spouses who were also involved with the learner's progress. Students left class smiling and clearly attitudes of fear had been replaced by pride in accomplishment.

As for my teaching style, I've become aware of the importance of letting the students "sweat" a little. When they have invested effort into the problem, the result becomes much more meaningful. Especially for adult students who are very anxious about math, I tried to intervene before their frustration level became too high. My students could choose not to return if they felt threatened and then nothing would be accomplished. I tried to make myself available as a guiding force for their exploration of math, but I have had to learn to back off and not be so quick to give them the answer but to question them in a way they can find the solution and validate their own work.

I intend to continue this type of procedure in my classes with some modification of exercises and continue to explore with my learners to help them acquire the number sense they need.

Of my fifteen students, Denise (my math phobic) received her GED with a math score of 46. She said for the first time in her life she didn't fear math and that it really wasn't so bad. Eight, including Laura, have been recommended for GED class in the Fall, and six will continue on the pre-GED level.

LEARNING TO LEARN. HOW LONG DOES IT TAKE?

By Leslie Arriola, Holyoke Adult Learning Opportunities (HALO) Center

In the adult basic education classroom, curriculum design must include approaches which allow the learner to explore and employ multiple strategies for solving problems.

(MASSACHUSETTS ADULT BASIC EDUCATION MATH STANDARDS, STANDARD 1: MATHEMATICS AS PROBLEM SOLVING)

The vision of mathematics learning in the *NCTM Curriculum and Evaluation Standards for School Mathematics* implies a significant departure from the traditional practices of mathematics teaching. Reaching for the goals articulated in the *Standards*—that students learn "to value the mathematical enterprise, develop mathematical habits of mind, and to understand and appreciate the role of mathematics in human affairs" (*Standards*, p. x)—means changing students' beliefs about what it means to learn and do mathematics. It also means changing the roles that teachers and students play in the learning process. Getting students to believe that doing mathematics involves reasoning, analyzing, problem solving and communication, not just computation and following alogorithms, is one part of the challenge of implementing this vision. The other part, the part that I invariably find holds up the process, is the challenge of getting students to take a far more active role in the way they go about the learning process itself—about learning to learn.

Learning to learn means taking the reins in the pursuit of knowledge, rather than waiting to be led. It means that, faced with a problem or a new concept, students will explore, question, hypothesize and struggle for understanding on their own, and that they will gain skills and confidence as mathematical investigators.

In my classes, I struggle constantly with students' resistance to becoming active, independent learners. I know this resistance is, in part, because I am asking them to play a role they do not understand or have experience in. But, it is more complicated than that. Take, for instance, the following situation.

Using fractions, Kelly is trying to figure out, how she spends her time on a normal weekday. On her own, she has made a list of activities with estimates of the hours per day she spends doing each. Looking over her shoulder, I see she has begun to translate hours into fractions:

sleeping - 7	7/24
working - 8	8/24
cook/eat - 3	3/24
driving - 1	
chores - 1-1/2	

Wondering what she will do with the 1-1/2 hours for chores, I sit down beside her and ask her to explain what she has done so far. She looks at me in a sudden panic: "This is all wrong, isn't it?" "How did you get the 7/24?" I ask, thinking that as she explains it to me, she will regain some confidence in her work. She searches my face for clues and says, "7 out of 24 hours...but now that I look at it, maybe it should be 7/17...24 minus 7 is 17." I keep quiet. She erases all the fractions and turns to me, defeated and expecting me to tell her what to do.

We've all been there. The minute we sit down with students, they stop thinking. It's an automatic reaction, a conditioned response: if the teacher asks you why you did something, it must be wrong. The answer is wrong, the reasoning is wrong, abandon ship — no matter how much sense it made before. Outside of school, Kelly is a decisive, confident problem solver, but when it comes to doing math, she has no confidence in her thinking. In part, Kelly, and the many ABE students like her, are running off of past failures in math. But the bigger part stems from the students' deep-seated belief that math students can only know what their teachers tell them to know, and math students can never know if they're right until a teacher says so.

We've all been there. The minute we sit down with students, they stop thinking. It's an automatic reaction...

Then there is the interaction between students' beliefs about mathematics and assumptions about the learning process, as this situation illustrates.

The GED math class has been making wall-size fraction bars out of 15-foot lengths of butcher paper, for a game we are inventing. They are working in pairs. The 1/3 and 1/5 people are moving right along, but the 1/4 and 1/6 folks have hit a snag: 4 and 6 do not divide evenly into the 15 feet. They don't know what the remainder means in terms of feet and inches. They ask me what to do. A great sidetrack, I'm thinking.

I ask them what they think a remainder means. They look at me blankly. I pull out the unifix cubes. Work with your partner, I say, and figure out how to show with the cubes 15 divided by 4 and 15 divided by 6. The room is suddenly silent except for the sound of each student wanly moving his or her cubes around on the table. After 30 seconds, even that sound is gone. I circulate, ask a few questions. It is obvious they don't know where to begin and it hasn't occurred to them to experiment. They are sitting next to their partners, but they aren't talking to each other.

Their dependency on me — the "authority" — is all too typical in ABE math classes. Adults who are normally exploratory and inventive in how they solve everyday problems, seem to leave these reasoning tools at the door of the mathematics classroom. Their view of math does not include invention and exploration. Math is cut and dried, something that has rules and methods that have to be memorized. And their view of math certainly doesn't include socializing. Talking to other students about how to do a problem means you're not smart enough to solve it on your own. If you're stumped, ask the teacher. If you want to learn something new, ask the teacher. If you want to know if you're right, ask the teacher.

Adults who are normally exploratory and inventive in how they solve everyday problems, seem to leave these reasoning tools at the door of the mathematics classroom. Their view of math does not include invention and exploration.

When I try to "teach" less (more open-ended questions, fewer answers, less lecturing, etc.) — because I want students to do more — they become confused and, often, panicked. They go blank. Over the years and with various groups of adult learners, I have tested with some success a variety of tactics to encourage students to take hold of the reins of their learning process and to change their beliefs that they are powerless as learners. But all too often, in the face of their confusion and passive blankness, I find myself back at the front of the class, trying to find a more comfortable launching pad for learning independence. And wondering what I could do differently.

LEARNING TO LEARN TAKES TIME

This project gave me the opportunity to take a closer look at how I can help students become more independent learners, how I can facilitate their transition from passive learners to active agents of their mathematical learning process.

Students' belief that math is a mystery on which only some authority can shed light is deeply entrenched. Changing this belief is not easy. Getting ABE adults to change the way they go about learning math is even harder. Both take time. But how much time?

Thus, this project was about taking time and tossing it out the window. Or, rather, gathering up all the time in the world and giving it to my math students. I wanted to give them a no-stress, no-tests, no-time-constraints learning environment, one in which they (and I) could take whatever time they needed to discover and develop their abilities to be active, independent, self-monitoring learners and knowers.

Knowing that weaning students from their dependency on me is a complex process, I explored a variety of methods to move us out of the traditional roles they expected and into new, more independent roles. I tried to go at their pace, not mine, in hopes that their resistance to change would fade pleasantly away, rather than having to be overcome. And I tried to make it all fun.

THE CLASS

My class for this project was made up of a small group of students recruited from traditionally-taught pre-GED and GED classes at a local adult learning center. These were students who wanted additional help with their math skills. A few of the students who signed up for the class had been in a previous "Math Fun" class I had taught at the center, but to most of them I was a stranger. Fortuitously, David, the director and head teacher at the center, became my partner in this project, not only in working with the class, but as a co-investigator and partner in learning.

The students were diverse in both cultural background and mathematical ability. After a few classes, some of the more advanced students opted to drop out, leaving a core group of six students. Each of the remaining students had had some experience learning math, but all of them felt that they did not have any real understanding of any of the most basic math concepts.

The class met twice a week: I taught the lead lesson on Wednesdays with support from David. On Mondays, David followed up on the lesson alone. Some students came to both classes, some to one or the other.

"MUCKING ABOUT" IN FRACTIONS

David and I wanted to focus on one mathematical concept and stay with it for as long as it took for students to truly get a solid grasp on it. We decided to center the lessons around a non-computational exploration of fraction concepts. Revisiting each concept often, but with different representations and in a variety of contexts, my goal was to immerse students in fraction explorations, instead of leading them through the usual sequence of fraction topics and computations.

In four months of fraction explorations, we:

✔ divided things up and put them back together again
✔ played with different referent wholes
✔ investigated improper fractions, proportions and fractions of fractions
✔ compared fractions of discrete and continuous quantities
✔ worked in both abstract and practical contexts

I asked them to work on fraction activities and problems in pairs and small groups, at their desks, at the board, around the room and even out in the parking lot. We made lots of manipulatives, invented games, made charts, drew diagrams, made up word problems, compared solutions, and talked. We talked a lot — to clarify, argue, question and pin point confusions. And we laughed a lot.

TEACHING IN PARTNERSHIP

My offer to teach at the center happily coincided with David's eagerness to find new and better ways to teach math to his students. Although he had some awareness of the ideas in the *Standards* and had learned a little about manipulatives and active learning methods at a few staff development math workshops, he taught math in the traditional way — by helping students work through a series of GED level math workbooks. He felt some students were learning math this way, but that "it was a drudge." It had also become clear to him that many of his students had deep-seated conceptual blocks for which the workbook approach was frustratingly inadequate. By observing me teach and collaborating with me, he hoped to gain new methods and a bigger framework to guide him in changing the way he teaches math.

For me, this added an unexpected and exciting dimension to teaching at the Center and to the project. Not only would I have the opportunity to implement the *Standards* in a situation free of time or curricular constraints, I would also have a chance to offer *Standards*-based perspectives and methods to another teacher. I hoped the spirit and philosophy of the *Standards* would provide David with the framework for change he so eagerly sought.

Thus our class would be a place where David, like his students, would have lots of time over the weeks we worked together to construct his own understanding and framework from what he observed — time to question and let ideas incubate, time to try out new ideas in class, and time to figure out which changes fit his personality and teaching style. Moreover, I would have feedback and questions from David to stimulate my thinking and teaching.

I also hoped David would discover that math classes can be as much an adventure for the teacher as it is for the learners, full of unknown paths, inviting sidetracks and chances to be creative, and that it's more than okay, it's important, for students to see their teacher take risks, make mistakes and admit that they don't know all the answers.

I also hoped David would discover that math classes can be as much an adventure for the teacher as it is for the learners, full of unknown paths, inviting sidetracks and chances to be creative

KEEPING TRACK OF THE PROCESS

It was (and continues to be) a wonderful teaching partnership. Every Wednesday, I'd invent an activity to start off the class, which I'd outline to David a few minutes before class. There was always a feeling of adventure as I began each class, partly because most of my starting points were just as new to me as they were to the class and partly because I knew I could never predict the students' willingness to participate.

During class, David took notes of what transpired: what I said and did; student responses, confusions and "aha" experiences; where we digressed or got bogged down. Also during class, he helped monitor small group activities and lent his good humor to the "fun" atmosphere I was trying to create. My copies of his notes, along with my lesson plans and own notes on each class, are the primary sources of data for this project.

After each class, David and I would talk about how it went: What were the high points? Were we finally seeing some signs of independent thinking? Benny was on a roll today. Who got lost? How can we keep Elaine on track? Why does Joanie refuse to work in groups? What did we (David and I) learn? What would be interesting for David to follow up with on Monday?

On Tuesday, David would call and tell me what he and the students did on Monday and how he felt about it. Taking my lead from what he reported, I would then quickly devise a plan for the class the next day. Sometimes there was continuity between our activities on the different days, sometimes we took off in different directions. The notes I took from his phone reports are also an important part of this research.

And, finally, I tape-recorded an interview with David at the end of the project in which I asked him to reflect on his teaching goals and methods, and how they had grown or changed during our partnership.

For me, David's notes and observations and collaboration are a gold mine. How wonderful to have another set of eyes and ears to help me see what went on during the class. How helpful to have someone who knows each student's background and problems help me interpret reactions and responses. What a treat to have someone to bounce ideas back and forth with.

WHAT DID I DISCOVER?

First, I discovered that, without the pressure of time and tests, students were quite happy to, as David put it, "swim around" and around and around, week after week, in non-computational fraction explorations. They took readily to hands-on learning: folding and cutting up squares and rectangles, exploring pattern blocks, making color-coded decks of fraction parts to "pack" and "unpack" into wholes and groups of wholes, finding the fractions in the room, in their homes, in the world.

Second, timelessness gave us space to unhurriedly sidetrack into gaps and confusions whenever they came up: What's a remainder? How can you divide a big number into a small one? How do you make out a check? How does addition relate to multiplication? Wonderful sidetracks, which more than once led us deep into investigations I would never have thought possible for this class.

One day, for example, I began the class by asking students to look around the room and find fractions.

Cass, grabbing a felt tip marker, said: "It's 1/8 red on the outside."

"How long is the red part?" I asked.

"The marker is 5" long," Thai said as he measured it.

"Then," Cass declared, "the red part is 1/8 inch long."

And off we went on a wonderful adventure to sort out the difference between 1/8 of an object and the actual measurement of 1/8 of 5". There was no way I could anticipate all of their gaps and confusions, but here we had the luxury of being able to take all of the time we needed to deal with them when they came up.

It was not long before it was clear they were really getting a feeling for fraction concepts. They were also becoming more comfortable with my untraditional manner and approach. They answered my questions more readily. They were less shy about going to the board. They talked more, though still mostly to me. They let me know when something suddenly made sense to them. And, of great interest to me, confidence ratings were higher. I often push students to commit to their answers by asking: "On a scale of 1 to 10, how sure are you of your answer?" They were beginning to know when they knew.

In fact, after just the third class, I wrote in my journal:

This was one of those perfect classes that was full of "teaching moments." The class went so well the students actually clapped at the end of it and many told me how clearly they had understood the concepts we were exploring — and how much fun it had been.

I confess to having had a wonderful time as comic orchestrator of the proceedings...I felt so "right on" the whole time...guiding the activities, drawing them out to conjecture, take risks, explain, make up questions....making it funny and fun. We were all motivated, engaged, energized.

"What a high! It can't get any better than this," I remember thinking as the class ended. It was only later, when I took a longer look at the day, that I reminded myself in my journal:

But.....this was a class with the teacher at the front and in charge, and the students still dependent on her to lead them.

I'm at home in front of the class. Like so many teachers, I'm a good explainer. I ask good questions. I can draw students out and keep them moving. I'm challenging, often funny. I was working hard for them and they were working hard for me. And that was the problem. They were still working for *me*.

As the weeks went by, it still seemed that without me to push and prod and orchestrate, they were lost. If, for example, I asked them to draw a picture of 12/5 of a candy bar, they would give up before they really tried and wait patiently for me to rescue them, no matter how long I waited them out. When I'd ask them to work in pairs, it was all I could do to get them to move closer to their partner, much less work on the problem together. They were lively, attentive and compliant — and still apparently totally dependent on me.

When they came up blank, was I jumping in too soon? Did I need better questions? How much should I let them struggle? How much could I expect them to discover on their own? How much did they need me?

I kept trying. I stayed alert for the smallest sign of independence and reinforced it. I moved them to one big table we could all fit around, which improved group discussions. I got better at devising activities in which they couldn't avoid working together. For example, asking them to measure the room with a long string meant they *had* to ask someone else to hold the other end and, giving each pair only one piece of paper to record their measurements forced them to work together to come up with a relative unit (e.g. one string length equals one arm span) of measurement. And I kept learning to reframe my questions. By the third month they were clearly more knowledgeable about fractions and were talking to each other a little more.

But were they more independent and self-reliant thinkers yet? At the time, I didn't think so. Then, as I read through David's and my notes, I found there were more signs of progress than I'd realized: Dave (one of the students) making up a new variation of our fraction game; Jackie monitoring and describing the way her mind works when she does math; Silvina realizing she needs to listen better if she wants to learn better; Silvina and Oeuy "arguing" over the way to approach a problem; the wonderfully inventive fraction treasure hunt they concocted (with David's guidance) for me.

I looked for more signs and — yes! — they were there. They were beginning to work longer at difficult problems. They were taking more risks in their efforts and were more confident when an effort made sense. On their own, they were noticing connections, seeing patterns, finding relationships. Another signpost I began to notice with great pleasure was that I was structuring the plans for each class less and less. That is, I could come in with just a starting point for investigation — for example, how to allocate the spaces in the center's parking lot (what fraction for students, for teachers, for deliveries, etc?) — and be reasonably confident that they would become engaged with each other in the discussion without a lot of lead-up activities led by me.

Music to my ears: David's report that, in his Monday class, students working in pairs to cost a recipe actually worked together. In my class, too, I realized there was no longer that passive, waiting silence when I left them on their own with a problem. Here and there I could hear the sounds of minds thinking and communicating about math. Now, when I said to them: " The answer is 7/16. Make up a question," they had a better idea of what to do. With less and less help, I was getting better and better questions. They were arguing with me now and then, explaining to each other, pointing out mistakes. Music to my ears, music to my ears.

Yes, I *am* doing less and the students are doing more. But we have just begun the shift. They *are* learning to learn, but progress is slow and these moments of independence and self-reliance are random and inconsistent. Yes, I'm teaching a little less (i.e I don't hear the sound of my voice quite as much), but there is still much to do to keep the process going.

As we continue to meet, the questions I ask myself as I plan for the class are more developmental: Where is the next challenge they can handle? How much longer should I leave them to struggle with a problem than I used to? How much more ambiguity can they deal with now? How much less do I have to spell out for them; how much are they ready to explore on their own?

Each week I remind myself that, because learning to learn is a slow, slow process, teaching less will also be a slow, slow process. Just because they are adults we cannot expect ABE students to become active learners the minute we give them active-learning experiences. People do not become analytical overnight. Beliefs do not change overnight. The ability to communicate reasoning doesn't come easily.

People do not become analytical overnight. Beliefs do not change overnight.

I have learned to pay closer attention to students' starting points. If they seem passive and dependent, I have found it is often because the idea of, for example, exploring mathematical relationships is meaningless to them. It has no basis in their experience. Nor can I expect them to defend their reasoning when they don't have the mathematical language to describe what they did. Some things I have found I must tell them or model for them. Others I can guide them towards. I'm more aware now that not all of the mathematical tools I want them to acquire can be discovered.

I am surer now that giving students this beginning support is not a sign of failure, on their part or mine. To the contrary, it simply signals the beginning state of a learning collaborative in transition. It is also a realistic indication of how long it may take to shift the expectations of students who complain: "Don't ask me to think. Just tell me the answer," from wanting answers from the teacher to demanding that they have a chance to make sense of a situation themselves. In time and with practice, lots of time and lots of practice, they will go looking for their own mathematical tools, new ways to approach, new ways to reason mathematically.

DAVID'S VIEW

Over the course of our partnership, David, too, has developed new tools and approaches for thinking about math and about mathematics learning, which in turn have given him the new framework for teaching math that he sought.

To begin with, his definition and understanding of math has changed dramatically. He said he used to view math as a linearly evolving sequence of computational topics— whole numbers leading to addition leading to subtraction leading to multiplication, and so on. Now David finds math to be a much more exciting subject.

I still see that certain mathematical ideas build on others, but now there are more dimensions. [Math is] a three-dimensional system of connections — a network of ideas intersecting and running parallel and next to each other.

Now that I've begun teaching math as thinking and problem solving, where computation doesn't have to happen, that opens up a whole new side of math to me. I'm learning a lot about the nature of math in a deep and essential way by teaching this way.

David's new orientation to teaching math comes as much from rekindling his excitement about his own mathematics learning as it does from new definitions.

You've paid attention to my own understanding of math ...encouraged me to do this problem, read this article. It's been fun for me. You've helped me revisit the feeling that I'm good at it. I like having that feeling. And that's certainly going to help my teaching.

You once said, and it's really stuck with me, "If you do math, you'll be a better teacher of math," which I didn't really buy in the beginning. Now I think that's very true. You can't stay very fresh in teaching math if you're not solving problems or challenging yourself.

In David's new framework, reasoning and problem solving have replaced computation in importance, and engaging students in mathematical inquiry and independent learning has become a new focus.

The major change in what I've learned is that computation can only go so far. Just focusing on computation is that old drudge. I've sort of taken it as a challenge to stay away from it, [to not be as] susceptible to falling into a computational mode. I thought students would demand it but that has not turned out to be the case.

The most interesting idea I've learned is that it's okay to raise questions and kind of leave them there, to throw things out without instant resolution, to put out questions and not have to provide answers.

He has begun to see the way in which group work facilitates mathematical learning and to understand the challenge of moving students into that mode.

It's good for people to be working together in a group context to solve problems. These students are unused to that and have needed to be brought out of their shells where they sit and do something on their own. It takes a lot of energy to get them to buy into a group activity, while at the same time checking in where each person is.

And, last but not least, David has discovered that "doing math" can be fun, socially and mentally — as he put it: "Math can be fun for fun's sake and fun in the thinking, and sometimes that's enough for the classroom."

Clearly, David has begun the transition to a *Standards*-based approach to teaching ABE math. It is also clear to me that helping David make this transition in the context of shared teaching made me as much a learner as he was. Together we explored new approaches to teaching and new ways to look at students' learning — sharing observations, comparing notes on how the students were responding, and devising new challenges for them. Articulating my orientation to teaching and learning math to David week after week forced me to reflect on how my own framework and beliefs were evolving. Thanks to our discussions, I am now much more in tune with my own thinking and teaching. Most importantly, through our teaching partnership, we gave each other support and encouragement to experiment with methods and approaches to make the spirit of the *Standards* come alive for our students.

MATHEMATICS AS REASONING

MANIPULATIVES VS. ROTE MEMORY

By Margaret Fallon ,Lawrence Adult Learning Center

Adult educators traditionally trained in imparting facts, rules, and direction are discovering that as facilitators and inquirers, guiding, directing, and encouraging students to question, probe, ponder, and devise and test possible solutions to everyday problems, they can enable these students not only to attain their educational goals more successfully, but also to apply their learning to real life situations more easily. Today, I no longer regard teaching as merely imparting knowledge or facts, but rather as co-directing critical thinking and creative discoveries in the classroom. Together, students and I can meet goals while sharing enthusiasm for the task at hand.

> ***"Students need to know that mathematics makes sense, is logical and even enjoyable... It is also essential (particularly for ABE learners) that concrete materials be made available to assist learners in supporting their reasoning, whether inductive, deductive, spatial, or visual."***
>
> MASSACHUSETTS ADULT BASIC EDUCATION MATH STANDARDS, STANDARD 3: MATHEMATICS AS REASONING

For the past fourteen years, my position as GED instructor at the Lawrence Adult Learning Center has given me the opportunity to work with many students at all educational levels who are from diverse backgrounds and cultures and who come to the Center hopeful that their educational needs can be met and their individual goals accomplished. During the past year (1992-1993), my involvement as a member of the Massachusetts ABE Math Team has highlighted the math component of adult education.

Although I have somewhat modified the question I explore here, it remains fundamentally the same one I have asked myself hundreds of times during my years as an ABE/ GED/ESL teacher. What will help math students who don't make progress because number facts are not at their fingertips? I have observed throughout these years that the majority of adults who failed to finish school for whatever reason come to adult learning centers with fairly good reading skills, but with limited math skills. I also have noted that poor memory of number facts, especially in multiplication and division, contributes greatly to further difficulty in subsequent math tasks with fractions, decimals, etc.

After serious thought I have concluded that, for reasons unknown to me, rote memory of number facts was simply not possible for most of these students, and, therefore, the emphasis of NCTM *Curriculum and Evaluation Standards for School Mathematics* on the use of manipulatives, calculators, and computer technology offers a new and challenging approach to math instruction for all adult learners, but especially for those who "just don't get it."

The question I have posed for my research is, "Will the use of manipulatives help students to better comprehend whole number grouping and lead subsequently to a better grasp of fraction concepts?" This question is indeed a challenging one for me, especially since all my math experience is rooted in rote memory of number facts, rules, procedures, paper-and-pencil computations, and one-way solutions with only one correct answer to any problem. Aided by a high interest in mathematics, I was able to complete all my math courses and to function successfully later on as a math teacher. Naturally, therefore, I assumed that anyone could be successful in math by concentrating mainly on the memorizing of facts and rules. How radically my thinking has changed! How clearly I now see the falseness of my assumptions! I shall describe my classroom environment, my students and current program, how I set about researching the question I posed, my findings, and my conclusion.

I assumed that anyone could be successful in math by concentrating mainly on the memorizing of facts and rules. How radically my thinking has changed!

Adult learning centers, though similar in many respects, have individual differences. The Adult Learning Center in which I work is located in a former primary school in South Lawrence and offers education opportunities to people of all abilities, backgrounds, and cultures from ages 16 to 75+. A wide variety of programs is offered to serve specific needs of a diverse student population which includes a high percentage of minorities.

To serve the needs of all students, classes at the center are held both day and evening, and students are given the opportunity to choose a program which meets their individual needs. All programs have attendance requirements, although the many and complex problems inherent in the life situations of adult learners often make it difficult to terminate a student because of poor attendance. In many such cases, a teacher needs the "Wisdom of Solomon" to make a reasonable decision.

My GED Preparation class consists of two groups of twenty students. Each student selects whether he or she will attend for six or twelve hours per week. One group attends Monday, Wednesday, and Friday; the other, Tuesday and Thursday. Hours are 8:30 a.m. to 12:30 p.m. Students are assigned to this class if their test scores according to the Adult Basic Learning Exam (ABLE) indicate a reading level of ninth grade or above. During class time students prepare for one or more of the five GED subject areas tests by working from commercially produced workbooks.

Because of the variety of goals of class members and limits on classroom space, individualized instruction is offered to help students attain personal goals in the desired GED subject area. This classroom structure has been effective and successful in the past. Now, however, I see a need for changing it in order to promote greater opportunity for group instruction and cooperative learning activities.

This need applies particularly in math and writing skills, but also in other subject areas. Although such a change was not possible during the last school year, I am optimistic about accomplishing it during the coming year. There is an urgent need for change and I would like to be the catalyst for its implementation.

Throughout this year, I carefully observed all of my students and tried new questioning techniques to foster the development of better critical thinking and problem-solving skills. I tried to talk less and listen more, becoming in the process a more reflective teacher.

My research project, as I mentioned earlier, has focused mainly on the question: "Will the use of manipulatives help students to better comprehend whole number groups and lead subsequently to a better grasp of fraction concepts?" For this report, therefore, I have spotlighted four students who typify the reason for this question. The table below shows their names, ages, last grade completed, and their reading and math scores.

In each case, there is a disparity between the reading and math scores, the minimum difference being four grade levels. It perplexes me that intelligent students can have such difficulty with rote math.

A brief profile of each student indicates good reading but poor math skills.

NAME	AGE	GRADE COMPLETED	READING SCORE	MATH SCORE
William	32	8	7.5	3.0
Roland	32	7	PHS	3.5
Marilyn	65	9	8.5	NA *Student unable to try because of anxiety*
Anthony	22	9	11	5.5

To gain further insight into the root of the difficulties, I interviewed each student alone, but present their answers together for the sake of comparison.

The questions I asked were as follows:

✓ How do you feel about math?

✓ Do you have a specific goal in learning math?

✓ Do you think that math is important in everyday life?

✓ Can you give me some examples of when you use math?

The questions ask how these students view math and what their past experiences have been. I anticipated hearing from some students who liked math but weren't successful at doing it and others who were frustrated by their inability to do math and who, therefore, hated it. I felt that hearing from them would help me to break the pattern of past negative experiences. I thought that if they verbalized their feelings, they might be more open to new ways of learning math.

Teacher: How do you feel about math?

William: Now you've done it. I can't do math. I can do everything but math.

Roland: I never could do math in school. That's why I quit school.

Marilyn: I think I'd like it if I could do it, but I don't know. I'll have to study my number facts during the summer. Do you think I can pass?

Anthony: You and I are going to have trouble with this one. I can't do fractions, and I don't know algebra either. And I forget, too. You like it, so it's easy for you. I hate math. I don't know why we have to do math tests anyway. I like to read better and you saw how I passed my social studies, science and literature tests. Why do we have to take fractions anyway? I can't write essays very well either, but I hate math. (Anthony is highly excitable, difficult to calm down, but he is challenging and I have enjoyed working with him.)

* * *

Teacher: Do you have a specific goal in learning math?

William: Well, if I could learn it, I wouldn't feel so stupid. My wife is smart in math. She can do division and even fractions.

Roland: I'm on disability because of a chronic back problem, and I need my GED in order to be retrained for another line of work.

Marilyn: I've always been interested in nutrition, but I could never get into college because I don't have a diploma. If I get my GED, I'd like to go to nutrition classes, and you need math there, too.

Anthony: I just want my GED, but I still wish I could get it without taking a math test. Why do we have to do math anyway?

* * *

Teacher: Do you think that math is important in everyday life? Can you give me some examples of when you use math?

William: I guess it's important, but my wife is smart in math and she does it all. I sometimes use numbers when I play cards with my father, but he thinks I'm stupid, so he always keeps score. He says I'll never learn.

Teacher: Do you think you'll never learn, William?

William: I'm learning now. One of these days he'll be surprised!

* * *

Roland: I know math is important, but I don't know my number facts very well, so I use a calculator when I go shopping with my wife. Sometimes I use math for measuring when I do things at home. I'd like to be better at math.

Marilyn: Oh, of course math is important in everyday life. I love to cook and I love to count calories. Changing recipes and adding up calories is math, I suppose, and I know how to do these. I measure material when I sew, too. I'd like to learn how to be better at comparing prices at the supermarket. My husband does that for me now.

Anthony: Well, I suppose it's important, but I just don't like it. I use a calculator sometimes, but I get mixed up with fractions. My grandmother helps me, but she makes mistakes, too. Mostly, I use math when I go to the store, but I'm not always sure I got the right change.

* * *

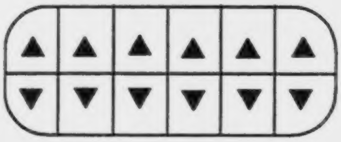
These students are typical of adult learners at many adult education centers in attitude and goals. William, for example, has a negative attitude. He's convinced that he can't do math at all. Roland says something similar: "I never could do math." Both could learn, but state they can't. Anthony expresses the frustration so many students feel. He has little patience and wants math to come to him easily Marilyn, on the other hand, thinks she might like math. She sees many possibilities for using math. She and Roland see the opportunity for new careers. For them, the GED is a necessary step to a larger goal whereas currently William and Anthony are focused on passing the GED and that's all.

When I told the students I was on the ABE Math Team, they were interested in the fact. I told them that math instruction is presented in different ways now and they might find it more interesting and even fun. They agreed to help me with my project and in the process they got excited about working with the manipulatives.

Initially, I worked individually with William because his math skills were so limited. He can count only by using his fingers or a tally. Using a chart, he has progressed slowly through addition, subtraction, and multiplication, but division has been overwhelming for him because the process is too complex. He seems not to comprehend the concept of dividing into groups. He has no idea of where, how, or why the numbers in a division problem go. He doesn't even recognize that the problem, when solved, is completed.

It occurred to me that manipulatives might enable William to understand the process of, first, dividing a total into equal groups and, second, determining how many different equal groups can be made from a given total.

A Dozen "Eggs"



The following account depicts William's progress in division employing manipulatives rather than rote memory. I began by using an egg carton containing 12 small green pattern blocks that I called "eggs."

Teacher: How many eggs are there, William?

William: I see 12.

Teacher: Yes. That is how they are packaged in the grocery store. Do you know another name for 12 eggs.

William: Eggs?

Teacher: Yes. That is how they are packaged in the grocery store. Do you know another name for 12 eggs?

William: Oh, you mean a dozen.

Teacher: Sure, Now look at the eggs in the carton. Can you tell me how they are arranged?

William: I don't know what you mean.

Teacher: How do they look in the carton?

William: You mean like rows?

Teacher: Very good. We could also say they're in groups. How many rows do you see?

William: (pointing) two.

Teacher: Good. Do you see another arrangement of rows?

William: I don't know what you mean.

Teacher: Well, you told me about the two rows across (pointing) six in each row, but can you look at it differently and see rows another way?

William: Do you mean up and down? There are only two eggs each time.

Teacher: Good. How many up-and-down rows are there?

William: (counting) Six.

Teacher: What can you tell me about two rows of six eggs across and six rows of two eggs up and down?

William: The number is the same.

Teacher: What number, William?

William: If you add these two (pointing across) and add all these (pointing), you get the same number: 12 eggs.

Teacher: Do you think there is another way of getting the answer?

William: Do you mean multiply? Like 6 x 2 or 2x6? I don't know multiplying very much.

I was thrilled that this exercise helped William grasp the concept of multiplication. He made the connection on his own that rows of equal numbers could be multiplied instead of counted.

After this relatively simple but essential exercise, I continued with a discussion of grouping. Referring to the egg box, I asked William to show me again the two equal groups of six and the six equal groups of two. Then I asked, "How many groups of two? How many groups of four? How many groups of 12? How many groups of three?" Although William had been having difficulty with comprehending what I meant by groups and grouping, he readily understood the concept as illustrated in this exercise.

I continued by asking, "How many groups of five? How many groups of seven?" This helped to clarify his understanding of equal groups and partial groups (remainders).

I then asked William to place 12 blocks (one dozen) on the table and count them aloud, first by twos, then threes, fours, and sixes. When I asked, "How many are in two groups of twos, two groups of threes, two groups of fours, etc., we shared a long, interesting, and active discussion in which William was highly motivated and responsive.

In subsequent sessions, we extended the activity using blocks, toothpicks, or pencils, increasing the total number gradually by twelves until we reached a total of 120. William willingly counted out and grouped elements, and, despite being unable to retain number facts in his head, he was able, by means of visualizing and manipulating, to comprehend grouping. Since rote memory of number facts and the proper placing of numbers in both multiplication and division problems present such a confusing and difficult picture for William, I have encouraged his use of a calculator as a necessary tool for his progress. We are ready to begin our work with fractions.

Many of the activities presented aimed at laying a foundation for understanding fraction concepts. Using

Cuisenaire rods, pattern blocks, and paper-folding, the students were able to create and discuss fractions of many different denominations. They were enthusiastic about trying any suggested activity, as the following shows.

Teacher: (showing Cuisenaire rods) Have you ever seen these before?

William: No, but I like the different colors.

Roland: No. What are they?

Marilyn: No. Are they for math?

Anthony: Yes, they were at school, but I didn't get to use them.

Teacher: They are called Cuisenaire rods, and they can be very helpful as math tools. As you look at them now, can you tell me something about them?

William:They are all the same shape.

Anthony: Not the same size, though.

Teacher: What do you say, Roland?

Roland: Each color must be a certain different size and the same color is the same size.

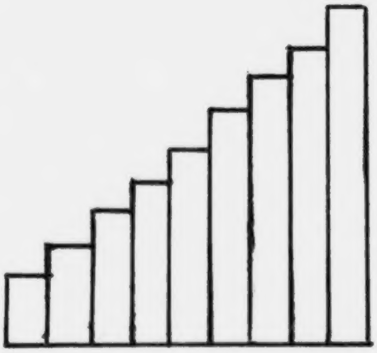
Marilyn: Oh, that's right. They are pretty. What do we do with them?

I thought it was interesting that Anthony had seen them but not used them. I could follow up in the future, but at the time, I wanted to move them into an activity.

For the rest of the time, William and Roland and Marilyn and Anthony worked together as partners. I posed the question: Let's say the orange rod has the value of one, one *whole*. Can you group parts of the same color to be equal to the orange rod?

William used different colors, two reds and one green, while Roland understood the instructions and used ten whites. I said, "William, your parts equal the orange rod, but each part must be of an equal size. Could you make a change and then have equal parts?

Cuisenaire Rods



William: Let me think for a while. I can't use another green because that will be too long. Oh, I know what you're saying now. I'll use five reds. There.

Teacher: Great. You both have made the fractional parts of a whole. How many parts in yours, Roland?

Roland: Ten.

Teacher: How many in yours, William?

William: Five.

Teacher: We discuss fractions as parts of a whole, so let's look at what you have.

* * *

Roland: They're both the same if you put the pieces together.

Teacher: So what can you say about that?

Roland: If the bottom and the top numbers are the same number, the rods are the same as one whole.

Teacher: Good. If I took these two, what fraction is that?

Roland: Two tenths.

Teacher: If I took two of yours, William, what fraction would remain?

William: I don't know what you mean.

Teacher: Roland, take two away from William. How many parts did you have at the beginning, William?

William: Five. Oh, now I see. Three fifths are left.

Teacher: Good. Now, looking at the rods again, do you think that any other same-colored rods are equal to the orange rod?

William: No. They are too big or too small.

Teacher: What do you think, Roland?

Roland: Most of them are too big, but maybe we should try the purple or the yellow or the green.

Teacher: Let me know what you find.

Roland and William could work well together because Roland caught on quickly and could help William even though Roland didn't have his number facts down cold.

Roland and William worked together for a while reviewing the 10/10, 2/10. 5/5, and 3/5. Trying the purple and green rods, they were able to determine that two yellow rods were the same as one orange rod. After they had lined up the yellow, red, and white rods and the orange rod, I asked if they could see any relationship between different-colored rods.

William: I don't know what you mean.

Teacher: Well, they all equal one whole: 10/10, 5/5, and 2/2

Roland: One yellow is the same as five whites.

Teacher: What does that tell you, Roland?

Roland: Well, 5/10 must be the same as 1/2.

William: Oh, I get it. Two whites are the same as one red.

Teacher: Good. William, can you say that using fractions?

William: Oh, I know what you're saying. There are five reds and ten whites, so 2/10 is the same as 1/5.

Teacher: Very good. We call 2/10 and 1/5 equivalent fractions. Equivalent means the same amount.

Anthony and Marilyn did the same activities, with similar results. All were interested, motivated, and responsive. My final interview question to them was, "How do you feel about math now?"

William: When I can see what I'm doing, it makes more sense to me. I understand better when I don't just see numbers.

Roland: This really helps me to see what reducing fractions means. I can see that 5/10 and 1/2 are the same. I like working with these things.

Marilyn: I think I'll like math more. It is so interesting to do things like this. Will we do more with blocks and rods next year? Maybe math won't be so hard for me.

Anthony: I like using rods and blocks, but we are just on fractions. Will we be able to learn decimals and algebra, too? I don't hate this kind of math. I just hate the math book problems.

In these responses, I noted that each student responded with hope about math instruction: "Maybe math won't be so hard"; "Will we be able to learn decimals and algebra, too?" The hesitant, fearful, frustrated comments from the initial interview were replaced by excitement about math instruction. The manipulatives would be a way for them to learn and to experience success. Their comments more than fulfilled my expectations.

I was sorry to see the school year end on June 30 because there were so many things I wanted to do with my students. It had taken me several weeks to obtain manipulatives. Then, by exploring their use, I was able to develop confidence in my ability to adapt my method of teaching. The change in my attitude was galvanic. I wanted to read, practice, and do more.

WHEN YOU HAVE A PROBLEM, USE YOUR HEAD (...AND YOUR INSTINCT)

By Thomson Macdonald, Haitian Multi-Service Center

In January, 1993, the members of the Massachusetts ABE Math Team attended a three-day workshop to immerse ourselves in issues of learning and teaching math. Among the many kernels of wisdom I collected in those three days about math and the world of learning, two underlying themes joined into an image which formed the basis for the project whose description follows. These themes have to do with first, a willingness and an interest in jumping into uncharted and uncertain territory, and second, a confidence that survival strategies will assert themselves to ensure a safe landing. The image for me is of a person at the edge of a deep and dark abyss who is contemplating a leap, and wondering about the possibility, once airborne, of successfully reaching solid ground. (This person could be he or she, but I will say she so that there is no danger that the person could be mistaken for me...) She jumps....

"In the adult basic education classroom, curriculum design must include approaches which emphasize mathematical reasoning so that the learner can draw logical conclusions from math situations using concrete models and verbal skills to explain their thinking."

MASSACHUSETTS ADULT BASIC EDUCATION MATH STANDARDS, STANDARD 3: MATHEMATICS AS REASONING

Most of the students in the learning center where I teach are Haitian. The math program was established three years ago in response to their demand for math instruction. To a great extent, I think this demand was fueled by a desire of the students to have a more complete experience of "going to school". They come from a tradition where the idea of one right answer is heavily reinforced, where knowledge is seen to come from the teacher, and where performance up to external standards is given a high premium. This is what they have expected in their math classes. Although I have an obligation as a teacher to meet my students' needs, I decided I would begin the winter session of the math class by trying to throw the students off balance. Their obvious delight in test-taking and worksheets reinforced my feeling that they depended on a style of learning which emphasized rote activities and was measured by correct answers alone rather than the reasoning implicit in arriving there. It is very often the case, in my experience, that students can the very next day not be able to do what they did the day before; this says to me that performance is given a higher premium than understanding in the students' view of math. I felt it was imperative that students lay aside their preconceived notions of math and math learning. The statements set forth in the the first four standards (Problem Solving, Communication, Reasoning, and Connections) in the *NCTM Curriculum and Evaluation Standards for School Mathematics* were, with my colleagues of the ABE Math Team, a source of inspiration in my attempt to foster new approaches to math in my class.

The class met an hour a day, four days a week. It was offered to prepare students for the EDP Math Diagnostic, a test students must pass (along with a reading and writing diagnostic) in order to enter into the External Diploma Program and eventually earn a Boston Public Schools high school diploma. While this test was recently changed to reflect an emphasis on reasoning as well as on computational skills, the preparation class could easily follow a standard progression through the basic math operations suggested by many textbooks. I set as my highest priority to create an atmosphere in which my students could engage their curiosity and have a sense of fun about their work in mathematics. My objective was to guide my students toward making their own discoveries and developing their own reasoning strategies through engaging in an active and personal process of problem solving. I hoped to encourage students away from the familiar approach of introducing a topic, presenting problems, solving them through paper and pencil computation, testing students' understanding, then moving to the next topic. The approach I took was controversial from my students' point of view: they were quick and eager to respond to any test-like activity presented in class, and ultimately they were there to prepare for a test which would give them entry into instruction at the high school diploma level.

I set as my highest priority to create an atmosphere in which my students could engage their curiosity and have a sense of fun about their work in mathematics.

In order both to keep track of the effect of my teaching approach and to coax students away from their habitual approach to math, I gave students questionnaires about their feelings about and past experiences in math, and I often asked them to write about the math class over the course of the session (with few results). In addition, I kept a sporadic log of activities and events which occurred in or around the class. I decided that it was essential that the class devote most of its time to non-computational math problems.

In the first questionnaire I passed around (Math makes me feel...), responses such as "...scared", or "...shame" were the rule rather than the exception; I knew little math could be learned in this environment. At the same time that I recognized my students' need for practice in computation and rote skills (the foundation of their experience in math learning), I felt that it was important that students not hang onto their pencils as if their ability to put numbers on paper was their only connection to math, and that some of their negative feelings might be allayed by deemphasizing numbers, operations, and the need for answers. I felt also that the non-computational nature of the activities I was going to introduce would present a positive challenge to the students' sense of what school is and what takes place in school, and to their sense of what mathematics is.

The class took on its jump into the abyss with two math activities whose challenges — if successfully met — might help us all to develop as mathematical athletes, secure in our ability to move, to react, to strategize and reassess, and ultimately to land on our feet. We took the logic problems called *MINDBENDERS* by Anita Harnedek, Critical Thinking Press and the game of visual perception called *SET* by Marsha J. Falco, 1991. I hoped that these would provide a structure, albeit unfamiliar, through which my students might begin to appropriate some of the authority traditionally vested in the teacher; that they might in each others' company realize their inherent ability to discover alternative ways into problem-solving; and ultimately that their sense of curiosity about math might be infused with a confidence which assured them that, having made the leap, they would eventually land on their feet. The following is a brief account of my experience with my class with these two activities.

We began with MINDBENDERS. At first sight, these problems do not appear to be math, if only because there are no numbers.

We began with *MINDBENDERS*. At first sight, these problems do not appear to be math, if only because there are no numbers. In retrospect, I think that my students were humoring me in agreeing to try to do these problems, but that they were waiting (less and less patiently) for the real math class to start. *MINDBENDERS* demand a knowledge of vocabulary and syntax, as well as some understanding of cultural context in some cases; more important, though, are the reading and critical thinking skills which they require. Each problem requires the solver to find a first step in, and I felt that before this step could be made, the solver had to be able to formulate an overview of the information given. I discovered in working with my class that looking for key words, a sort of scanning exercise, was critical to this formulation, as were locating and organizing a progression of clues. The *MINDBENDERS* book offered a grid to use for a process of elimination whereby the various pieces of information could be matched. I did not offer this grid right away because I felt that it narrowed the approach to a specific structure, and my purpose was to encourage students to develop their own structures. I urged the students to draw pictures or diagrams, make lists, and discuss among themselves as they searched for a foothold. I repeatedly asked them if they thought this was math, and why. Some of my students drew a complete blank on the problems: they didn't have any idea of how to get into the problem, much less an idea of how to begin a process for solving it.

Confusion, frustration, and disinterest often threatened to prevail in work on these problems - though there was the infrequent counterpoint of demystification and comprehension. I did not feel that the students in my class were ever fully together in working on these problems. Class momentum came and went. I would revert to worksheets and problems with clear computational goals, then return to *MINDBENDERS*, hoping that the students would plant their feet firmly inside the problem. I tried to have the students work in small groups so that they could begin to break their habit of approaching the solving of math problems in solitude, so that they could develop together a process of reasoning and elimination, and so that they could share in the excitement and satisfaction of getting to the solutions. With *MINDBENDERS* I could tell that a group was on the trail of something first when they spoke at all to each other, and then especially when they began to speak in both English and Kreyol. One day this problem was presented:

Four children (Hisashi, Marguerita, Nuhad, Phoebe) are playing hide-and-go-seek at Hisashi's house. One of the children is "it". The other three are hiding in three different places - behind the garage, behind the house, and behind a tree.

1. From where he is, Hisashi can see the girl who is hiding behind the house.

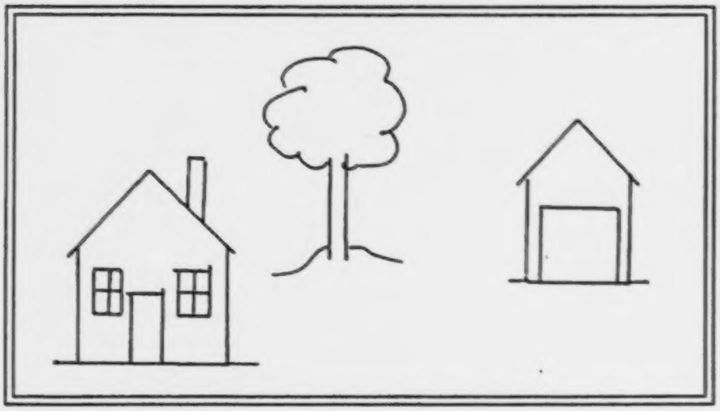
2. Nuhad can't see the garage from his position.

3. From where she's hiding, Phoebe can see the girl who is "it".

Who is "it"?

Where is each child hiding?

One student in particular had a very difficult time even beginning to unravel the problem. He did, however, very carefully draw the garage, the house, and the tree with a ruler. He and I were both pleased with his diagram, but I was concerned that he was stymied in putting a child's name to each position, and concerned, too, that he was working in isolation. Another group of three had found their way into the problem and slowly worked their way to a solution. They were excited by their process, and were able to find the answers. When they finished, I asked the isolated student to join them so that they could

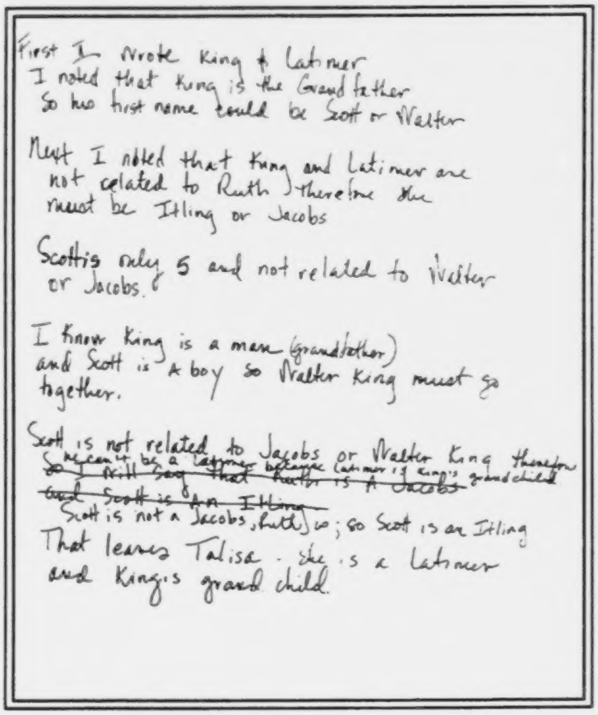


explain their solution to him. He listened to them with the same sort of blankly puzzled expression he had shown once his diagram was drawn, until finally he latched on to a piece of the explanation being given to him. As he began to understand how the clues fell into place for this problem, a smile slowly lit up his face and I could quite literally see math wheels begin to turn in him.

This was certainly a moment of success for me. In the first place, this student had begun to understand a logical progression of steps and how they led to a solution. In the second place, and just as important, it was his fellow students who made this understanding possible for him: they had been able to explain their solution by way of a step by step process of discovery, reasoning, and elimination. On that day I was happy to see students working with and explaining to each other, using their pencils to point to words or to pictures as they spoke, showing excitement about their discoveries.

This moment of clarity was, unfortunately, one of only a few times that I felt with *MINDBENDERS* that the class moved successfully and willingly away from the expectation and need for the rote computation and worksheet format. The problems continued to be more bewildering and frustrating than challenging and rewarding for most of the students. In an effort to evaluate the value of continuing with *MINDBENDERS*, I met with the EDP reading and writing teacher several times. She told me that if she encountered problems such as these on her own, her immediate reaction would be to close the book. In working with me and with the students in her class, however, she has become fascinated by the reading, comprehension, and critical thinking skills which *MINDBENDERS* address.

One day I showed her a problem I had had some difficulty in solving. She was able to match the first to the last names in this problem very quickly, and I asked her what was the first name she was able to identify. She sketched out her thinking process very briefly, after which I asked her to write out her full solution to the problem, that is, the step-by-step reasoning process she went through to arrive at the solution. When I read her explanation, one step in her process was unclear to me, and I told her where she had lost me. She agreed that that part of her reasoning was not clearly articulated, and rewrote it. In talking about the process of problem-solving we had just gone through, we realized how useful *MINDBENDERS* can be in integrating math skills with reading and writing skills. In doing the problem, Marsha reacted with the sense of fascination I had hoped my students would show. Though my students had only occasionally been able to explain their solutions (and never clearly in writing), through working with Marsha I was able to reassure myself about the usefulness of the class' explorations with *MINDBENDERS*.



First I wrote King & Latimer
I noted that King is the Grandfather
So his first name could be Scott or Walter

Next I noted that King and Latimer are
not related to Ruth therefore she
must be Itling or Jacobs

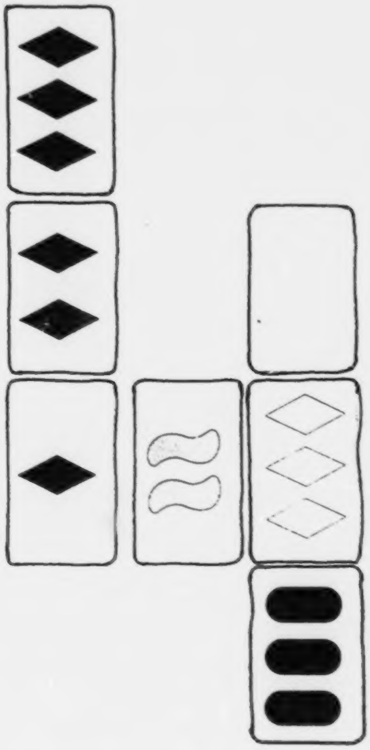
Scott is only 5 and not related to Walter
or Jacobs.

I know King is a man (grandfather)
and Scott is A boy so Walter King must go
together.

Scott is not related to Jacobs or Walter King therefore
he can't be a ~~Latimer~~ because Latimer is King's grandchild
~~So I will say that Ruth is A Jacobs~~
~~and Scott is An Itling~~
Scott is not a Jacobs, Ruth is; so Scott is an Itling
That leaves Talisa - she is a Latimer
and King's grand child.

Having worked with the students at some length in the *MINDBENDERS* realm, I finally got myself to the store to by a game of *SET*. I was excited because I thought the students would see in this game a much more familiar connection with their world of math — even though, like *MINDBENDERS*, it does not require the use of numbers, and, in fact, doesn't even have any written words. What I liked about the *SET* game is its deceptive simplicity. The instructions suggest a very concise rule of thumb: "If two are... and one isn't, it isn't a *SET*". I had in mind for my students to work on recognizing "same" and "different", on understanding how "same" and "different" can be combined, on recognizing and applying the concept of properties, and on articulating their reasons for the choices they would make in the game. We had been working on fractions — not operations with fractions but simply an understanding of what a fraction is: "What are the properties of a fraction?" "How do pieces go together to make a whole?" "What is a whole?" How do we identify and name the pieces and the whole?"

The *SET* game seemed to speak to many of these questions, albeit in a non-computational context. There are four properties presented on any one card: color, number, shading, and shape. In a certain context, *SET* is a competition in which a number of cards are laid out and the object is to be the first to identify a set. I was interested in approaching the game as a type of exploration, to have students make sets as a cooperative activity, and then be able to clearly explain why a group of cards could be designated as a set. Like *MINDBENDERS*, *SET* is an excellent vehicle for language practice as well as development of critical thinking skills. I was particularly intrigued with the possibility of having students describe what sets could be possible given one card — they would be challenged to imagine what two remaining cards would complete the set, and if there might be more than one possible solution. While in this case there is a finite number of answers, the fact that there is more than one can encourage students to explore without the sense of fear or shame they had expressed about their math abilities.

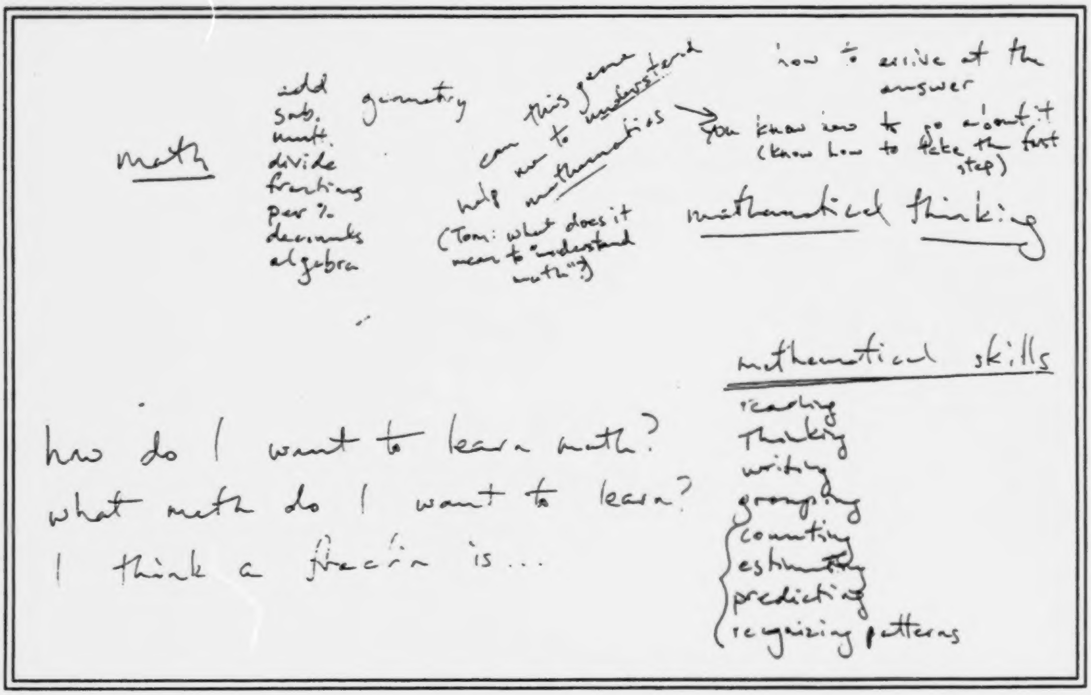


SET © 1991 Marsha J. Falco

On the first day (as suggested in the instructions for the game) I introduced *SET* using only red cards, thereby reducing the number of properties under consideration to three. It was a day of questions and explanations. The next day the class split into three small groups, each of which had cards of one color and I asked each group to arrange the cards in sets (interestingly, all three groups independently decided on same shading, same shape, different number as their definition of a set). To encourage them to find other sets, I suggested that they make intersecting sets, like a crossword puzzle. On the third day, I arrived to an announcement from a student in the center's Adult Education Program who also serves in a staff role: "You have a problem with the math class. The students think the math is too light."

This brought activities with the *SET* game to a complete halt, and made starkly clear just how far into the abyss I had fallen. I wondered if any real teaching had taken place in my class since it began in January and I walked into the classroom that day with genuine trepidation. I asked the students what their objection to *SET* was; I held the game in one hand and a fraction book in the the other, and asked which they preferred, and why. What I had felt as I had brought worksheets from this book to our previous classes about fractions is the relief and enthusiasm the students showed for this familiar approach to math — many of them even wanted to take worksheets home as homework. What ensued was a broad ranging discussion about math, as we began to talk about what distinction there might be between mathematical operations and mathematical thinking. The students first collectively created a list of the math operations they knew of, and then we moved to the question of mathematical thinking. One student asked: "Can this game help me to understand mathematics?" My response was another question: "What does it mean to understand math?" Answers included: "...how to arrive at the answer...know how to go about it...know how to take the first step..." Finally the class and I together made a list of what we thought mathematical skills were (e.g. reading, thinking,counting....). Before the class ended I asked the students to write what they thought of *SET*. Responses varied from: "Yes, the game is good but when are we going to do math?" to "This card game does not belong in school for adults and is a waste of valuable time."

I don't know yet if using the *SET* game helped to develop my students' understanding of fractions or ultimately served to strengthen their ability to use fractions. I do know, though, that the discussion we held that day was frank and engaging, and that the central topic was math. That students who began the class with no thought of math as a context for opinion and discus-



math

add
sub.
mult.
divide
fractions
per %
decimals
algebra

geometry

can this game help me to understand mathematics → you know how to go about it (know how to take the first step)

(Tom: what does it mean to "understand math"?)

how to arrive at the answer

mathematical thinking

mathematical skills

reading
Thinking
writing
grouping
counting
estimating
predicting
recognizing patterns

how do I want to learn math?
what math do I want to learn?
I think a fraction is . . .

sion had reached this moment of articulate perspective about their class was a significant moment for me as a teacher. I took great satisfaction in the writing that students composed in reaction to this day of discussion. After weeks of getting little or no writing from them either directly about math or about the learning process of the class, despite repeated urging on my part, finally the students wrote honestly and directly about their class and about math. While what the students wrote was not universally positive, these following words gave me reassurance that at least one student had moved away from a notion of math as a rote and isolated activity:

When I first began doing Basic Math with my teacher, I disliked going to the board. Since I had no self-confidence, I thought my fellow classmates would laugh at me if I made mistakes.

One day my teacher suggested that should work in groups rather than by ourselves. This has helped me in that we were able to share our different ideas, and how to find a sloution to the problem. Working together has not only helped me at school but at work. Now I am more involved and outspoken. Some of my co-workders wonder at times if I am the same person that used to sit so quietly without saying anything.

I want to encourage my fellow classmate A. who is having this problem of inconfidence don't be discouraged. I was like him some time ago but I have overcome through working in groups. My mind is at ease because now I know the students and they know me. I enjoy working together with them. Because of this, I am able to concentrate on what is being taught.

Student

The question always comes up for teachers and students whether they can afford the time for such a discussion when it comes at the expense of a day or days working on math basics in preparation for one or another test. Will non-computational math activities like *MINDBENDERS* and *SET* help students to pass a test? I have my own beliefs about this, but no irrefutable research to support them.Of eight students from the class that took the EDP Math Diagnostic some weeks after they left the class, six passed. While this is a fact which is gratifying to me, it is by no means hard evidence that the nature of the class was directly responsible for their success. I wonder still, as I did the day the students questioned *SET*, what teaching and learning had happened over the weeks the students and I had worked together. Looking back, I ask myself if I and my students would have been better off if there had been a better compromise of the non-computational activities I was eager to challenge the students with and the worksheets which the students seemed to need for their sense of balance. I jumped, and am wondering where I still will land, but I can say that my convictions remain firm that a connection with math is not so much a body of discrete operations computed with pencil and paper as it is a mode of thinking and acting, and that is my responsibility as a teacher to bring these convictions to my classroom. What I want for my students is that they be curious about approaching the abyss, about where they might land if they jumped, and that they be confident that wherever they land, it will be on two feet, skillfully.

"THEY NEVER ASKED ME. WHO DID THEY ASK?"

By Shelley Bourgeois, Jackson-Mann Community Center and Martha Merson, Adult Literacy Resource Institute

"The secret language of statistics, so appealing in a fact-minded culture, is employed to sensationalize, inflate, confuse, and oversimplify."[1]

During the Fall of 1992, we co-taught a math class to adults at the beginning levels of reading and math. Shelley is the constant teacher, and Martha worked with Shelley until June, 1993. Shelley also taught this same group reading and writing. Shelley brought in an article from the *Boston Globe* about advertising that focused on the symbols of advertising that are racist. She was amazed that a student, who was a daughter of a sharecropper, could not make the connection that the image of Aunt Jemima used to promote maple syrup also promotes racial stereotypes. Beyond this one disturbing example, the class as a whole tended to accept the images and information reported in the newspaper or by the media in general as truth. They had no concept that they were represented in statistics and those statistics affected their daily lives. We set a goal for the spring to encourage students to question numbers when they came across them in articles. Our long-term goal, however, was for them to spontaneously question all information they come across. To accomplish this goal, we chose to introduce students to data as it is displayed in numbers in graphs or tables.

> ***"Adults are bombarded daily with results from statistical studies that can and do impact their lives.***
>
> ***Curriculum design must include approaches to teaching statistics and probability which allow the learner to evaluate arguments that are based on data analysis."***
>
> MASSACHUSETTS ADULT BASIC EDUCATION MATH STANDARDS, STANDARD 9: STATISTICS AND PROBABILITY

In this paper, we report on and analyze what happened during the spring in an attempt to answer:
Will learning about statistics and probability enable my students to view their world more critically and encourage them to ask more questions?

CONTEXT

This is a multi-level class with levels ranging from Beginning to Intermediate. The students represent El Salvador, Haiti, Laos, Jamaica, and the United States. All except one student lives at or near the basic poverty level. The class has a core of students who have been members for five or more years. Shelley began teaching the class in September, 1992. They had their previous teacher for five years.

The members of the class are very supportive of each other. They celebrate EVERYTHING — birthdays, holidays, etc. Class is very much a social gathering as much as it is an educational one. It isn't that they won't do what's asked of them, but they need their time, too.

The following descriptions are of four students who will serve as case studies. They are from different backgrounds and have very different learning styles. The case studies show how they reacted to the process. Each typifies or exemplifies some of the different learning styles in an ABE/ESL classroom.

Ethel. 64 year old African-American woman with severe dyslexia. She is an extremely self-motivated individual who has taught herself how to survive in the world. She was abandoned in a strange city at the age of nine. After twenty years of tutoring, she can read at a third grade level. Her reasoning skills are excellent, and she brings her life experience into her math experience. She has had to fight for everything she has gotten in life. She questions almost everything that is told to her. Her goal is to learn. She has been in the class since September, 1992.

Mabel. Middle-aged African-American woman who changed her age every time we used ages in class. Mabel was born in Georgia, a daughter of sharecroppers. She went through the fourth grade, but her attendance was sporadic due to harvest seasons. She is a very cautious person. She likes to be treated as an adult. "No blocks!" (manipulatives). She is more likely to ask another student for help before asking a teacher. It is hard to know when she really understands something because she has a difficult time verbalizing what she does and doesn't understand. She has been in class since 1987.

Robert. 37 year old African-American man. He suffered a head injury which doesn't impair his ability to learn. He lives in a half-way house for recovering substance abusers. He has been in recovery for one year. He reads on an Intermediate level. His reasoning skills are excellent even though he reports that he is a poor math student. He is friendly with his classmates even though he is very shy. He doesn't share much about himself. His goal is to get his GED. He has been a member of the class since January, 1993.

Anna. Middle-aged woman from El Salvador. She has been a member of the class since 1987. She had very little formal education and is very nervous about learning new things. Anna prefers us to give her worksheets to math activities. She doesn't give credit to the knowledge she has obtained from experience. "You have to do it in the book, and then get it checked by the teacher." Anna is now an Intermediate level reader.

BASIC OVERVIEW OF 3-HOUR MATH CLASS.

Mental Math. Mental math is a set of ten questions in which we ask the student the question, and they have to work it out in their heads.

Some kind of activity.

Some kind of game both for reasons of attention level and because they liked them and seemed to get something out of them.

Some kind of vocabulary.

WHAT WE DID

To teach statistics, we used graphs and data.

Whenever a graph was presented to students, they had no idea where to begin interpreting it. They did not know to look for the titles, source, range, or how to interpret the information.

We didn't do formal vocabulary of statistics and percentages with the class. Instead, we encouraged them to notice what the graphic was trying to tell them and to manipulate the numbers to see what the results would be. The skills of estimation, number sense, and reasoning were encouraged. This class prefers real life manipulatives so our resources consisted of articles and manipulatives that are familiar.

We will examine four memorable classes that occurred in chronological order. We will explain what happened during the lesson, what made it memorable, and how the students responded.

1. Who has Sedentary Lifestyles?

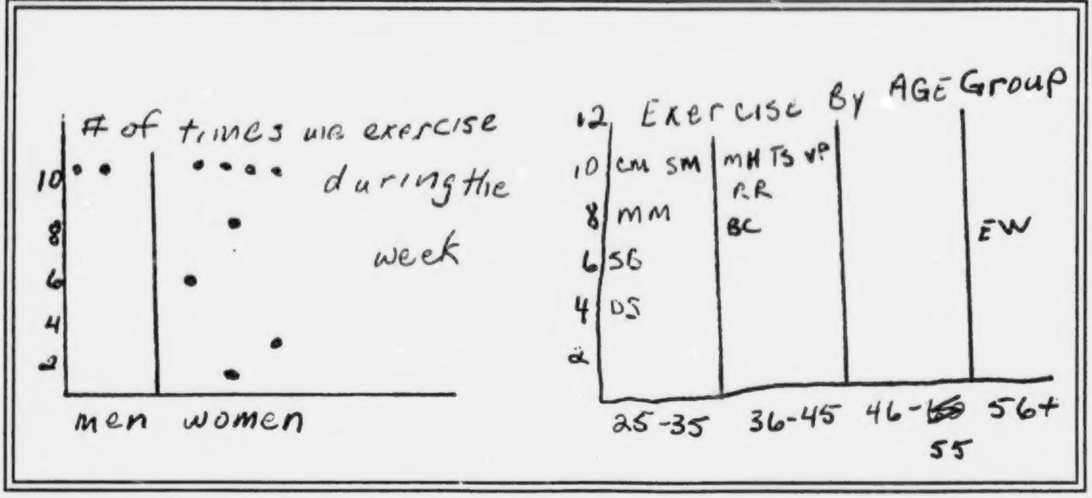
In March, Shelley brought in a graph from an article entitled "We Aren't as Fit as We Think," by Richard Saltus found in the *Boston Globe*[2]. The source of the graph was the Massachusetts Department of Public Health, 1990 figures. It examined the Healthy People 2000 campaign which plans to reduce the number of Americans who are defined as medically "sedentary" — no leisure time exercise. There were two graphs represented. One related the national, Massachusetts, and Year 2000 goal percentages. The other reported the percentages for the state's most sedentary — those 65 and older, blacks, those with low income, and Hispanic women.

The students were presented with the graph. Since up until this point the class had not responded to information displayed graphically or in articles, my main goal for the lesson was to have them identify the title, ranges (without defining it), and source, and to understand what the graph was trying to tell us.

The class exploded! Evelyn in particular was angered by this graph. She said, "They never asked me. Who did they ask?" This prompted a discussion of population and sampling. Another student pronounced, "They are always putting black people down." Ethel, too, was upset about the label "Blacks." She said, "They don't know what black people do in their houses. If they know what's going on, why don't they help us?"

The class went on to notice that most minority groups are treated in similar ways. They discussed which immigrants try to pass for whites and which ones cannot.

The class decided to make their own graph using themselves as the population to see if they could disprove the *Boston Globe* graph. Our first step was to plot each student and his or her own level of exercise. At this point, it became clear that we needed a consistent definition of exercise. Would running up and down the stairs count? Would cooking count? Or would only intentional exercise count, like a trip to the mall for the purpose of walking. Once we had a graph with individual information, we then asked the class how we could group the information. They concentrated on the categories of Sex and Age Ranges, and refused to do a graph based on nationality.



BEST COPY AVAILABLE

When we were constructing the graph on Age, Ethel asked where 50 was. This prompted a discussion on ranges. The class decided to change the range categories to make them more reflective of the class. They became 25-35, 26-45, 46-55, and 56+ instead of 25-35, 36-45, and 45-60. This change made the categories reflect the class better. Martha believes some students were confused when the range stretched from 45-60 because the "50's were invisible."

The vocabulary words we had used during class we defined and reviewed again at the end of the day. They were *data*, *hypothesis*, *sampling*, and *categories*.

This day was important for three reasons. First, it was a highly emotional discussion stimulated by a graph. The numbers meant something. The class realized that they were being represented by those numbers. Second, Ethel was able to quickly question the basis of the numbers. She modeled for others appropriate questions to ask about a graph: Where did this information come from? How many people did they ask? Why don't they do something about it if it's true? Third, this discussion became a touchstone for the class. Whenever we needed an example of a hypothesis or sampling, we could refer back to the graphs we had made about exercise. We were sure that everyone who had been there would remember the discussion.

While Ethel continues to have difficulty with computational math, during this class, she distinguished herself as exceptionally competent in assimilating, questioning, and reasoning when dealing with numbers presented in graphs.

We knew how important it is to present students with relevant, familiar information. At the same time, we wanted to introduce a new concept. This day really brought home to Shelley in particular how far learning can go when students have a vested interest in the subject.

2. Raisins Per Box - What's Your Guess?

The students had experienced how powerful graphs could be. After the Sedentary Lifestyles lesson, we did similar activities with graphing shoe size, family size, Martha's grocery bill, but we wanted students to be more engaged with the whole process of graphing from hypothesis formation to graphing and analyzing the results.

We wanted students to be more engaged with the whole process of graphing from hypothesis formation to graphing and analyzing the results.

Shelley found this lesson in a book called *Used Numbers Statistics: Middles, Means, and In-Betweens.* She chose the lesson "Raisins and More Raisins"[3] because it was the least childish, could be completed in a three-hour class period and included all the research steps yet was contained in the classroom.

As we handed every student a small box of raisins, we asked each person to estimate how many raisins were in the box. We wanted to record the guesses in a graph, so as a group we decided on the range for the vertical axis. Then students recorded their guesses on a table on the board, locating points by tracing up from their initials and over from the right number. Then each student had to count their box of raisins, without eating any. We recorded each person's actual amount next to the estimated amount. We examined which student got closest and talked about why some boxes had fewer.

Anna had the closest guess, and Mabel's was the furthest off (by 100 raisins). In this situation, a student could easily have felt embarrassed to be so far off, but in this class, students took everyone's guess as a valid, serious guess. Nevertheless, Anna felt proud, and we reinforced her excellent estimation by repeating that she came the closest. From then on, whenever we talked about estimation vs. accuracy, we said, "like the day Anna guessed how many raisins and then we counted and found out how close she was."

Beyond the idea of estimation, we worked on the mathematical concepts of averages and direct and indirect relationships. Robert benefitted from the sequential nature of the raisin activity. He moved through the steps smoothly, understanding the logic behind each. He grasped the idea of average easily and was able to use it during the rest of the classes. Yet later when we asked the students to define the words in writing, they still had difficulty articulating a definition. Robert said, "Numbers on weight, height, or amounts." Anna gave examples of its use such as: money — she matched this with food; bills — she matched this with work, and height — for a person or building.

The day's words were *average, data, direct and indirect relationship*, and *sample*.

At first, we did not realize how memorable this lesson was. Shelley thought it was tedious, and thought it hadn't gone well because moving students through the steps felt like pulling a camel. The students found the lesson to be more memorable. We believe, in part, it had to do with the edible nature of raisins. We found that the raisins helped set apart one math class from another. They recalled this class fondly when asked to evaluate the spring cycle.

3. Bagels - What Tickles Your Tastebuds?

Martha brought in a graph found in the *Boston Globe* Fun Pages on bagels.[4] A group of 172 kids and teachers had been polled to find out what their favorite bagel is. We used this graph to get away from bar graphs. We knew the raisins had been successful and had a hunch the bagels would be, too. Many of the students had never had a bagel with cream cheese before.

We discussed what a survey is, then they each got to try a piece of the three types of bagels we had. Ethel asked, "How may people need to be interviewed before you can draw a conclusion?" The class was split into two teacher-designated teams because some peer tutoring was developing into dependency. Unlike the groups that existed before this unit, these groups were heterogeneous by level. Each group made a survey of itself which they transferred into graphs based on the model they had been given. Even though the groups had big pieces of newsprint and markers, they drew small circles. They didn't take many risks when they drew. Instead, they kept close to the model, a fairly small bagel-shaped circle graph. The words for the day were *survey*, *data*, and *population*.

Robert was the only one who converted the information into a bar graph. He actually did two bar graphs. One was perfect; the other had a problem with direction. His circle graph did not represent the information correctly. His division of the circle didn't correspond to the numbers. The sector with four had the same amount of space as the sector representing one person. He adopted the % sign from the original, but placed it after the actual numbers.

Mabel had a hard time applying information from the bagel graph. What would it mean to a small storekeeper who sold bagels that so many people like plain and so few like sesame seed? The question wasn't making sense to her. Should he keep the same number of all kinds on hand? Martha was surprised that Mabel could grasp the idea that the bigger the raisins the fewer in the box but couldn't generalize the cause and effect relationship in this scenario. Mabel's problem keeping the connection between bagels and sales worried Martha.

This lesson worked for this class because it built on the class culture: celebrating with food. This class with its Laotian, Haitian, Jamaican, and Latino participants wasn't shy about trying new foods. For Martha, the role of teacher as corrector became more burdensome in this class. Even though the students were in teams, they needed us to check their graphs and provide hints for correcting mistakes. It was at this point that she realized one way out would be to ask students to do graphs incorrectly. The next assignment could be done in groups again.

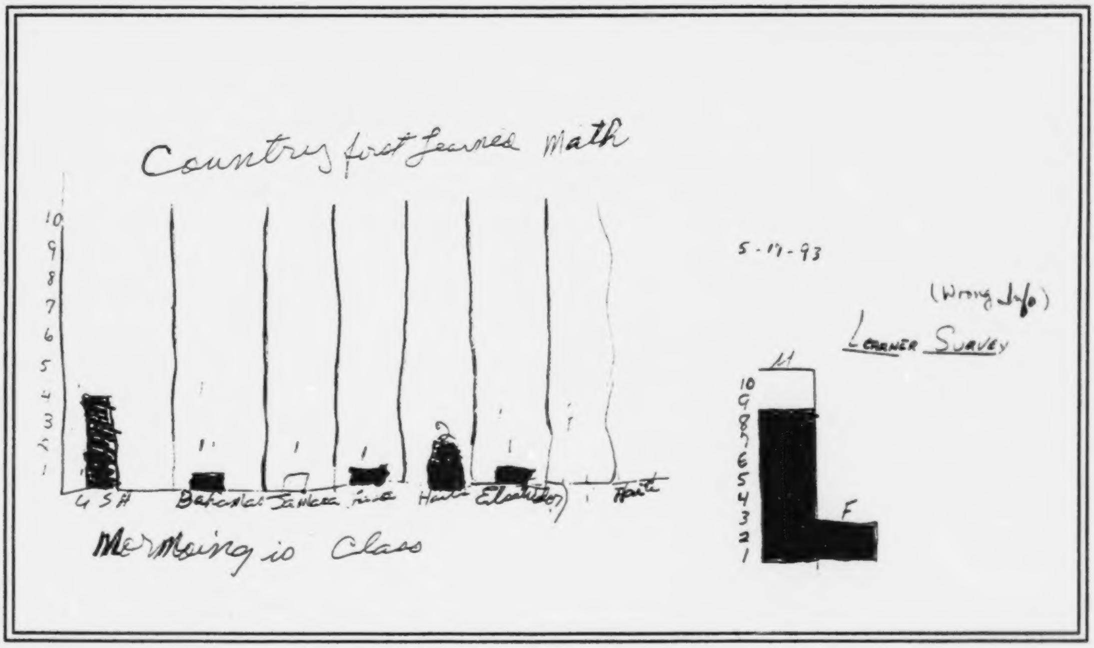
4. Bonnie and the Research Project

Another project the math class did was connected to a larger Massachusetts project: Research into Adult Basic Education Mathematics (RABEM). The primary researcher, Bonnie Mullinix, spent two days with the class during which the students acted as consultants. They helped her design the interview questions for other ABE learners. An example of a recommendation was to take out the word "manipulatives" and to ask "What are the different things (objects, materials) that your teacher uses to help you learn math?"

Some of the students really enjoyed the process. On the other hand, some students saw no relevance to math class. Anna left early the day Bonnie visited. She didn't think this was math. Mabel, Ethel, and Robert, however, were comfortable with the process. They understood that Bonnie would use their input and that the wording of a question can affect the outcome.

We wanted to incorporate the RABEM project into the class, but because of the different time frames, we weren't completely successful. The learners didn't get to see the data from the other learner interviews, but the whole class filled out the cover sheet from the learner interviews and split into groups for the false graph activity. The groups would have to do graphs with all kinds of mistakes in the title, source, ranges, as well as in the actual numbers on the graphs. Then it would be up to the other group to find the errors and correct them. In asking students to do a graph wrong, we would give them another chance to work with the parts of a graphic representation and find out whether they knew all the parts that ought to be included.

When we did this assignment, we modeled a wrong graph with information about them. Our false graph stated they were an evening class and had twelve participants from Egypt. The groups then worked on similar graphs for each other. This brought out playfulness and reconnected them strongly to what they were graphing. They worked in groups for the rest of the cycle.



The two small groups chose different strategies to accomplish the task. One group made an accurate graph of the learners' countries of origin first, pictured above. The other group launched directly into a graph of learners' genders. In fact, the class was dominated by women, eight to three.

These graphs were the best of the Spring Cycle and represented a compete understanding of setting up two axes, translating information to points on a bar graph, and interpreting the information.

Because Shelley realized she could use this as a final assessment, she didn't introduce any new words.

OUTCOMES

We did not have a set agenda for the students to adhere to, nor did we have a defined set of outcomes that we wanted to happen. As we went along, we developed a clearer sense of what we wanted them to take away with them.

We didn't care so much that they learn vocabulary and formulas, but that they not take everything that is told them for granted as truth. We know that although we reviewed the definition of data in other words, this is not something we could rely on them remembering. Similarly, the concept of sampling (how 500 people can stand in for 500,000) was only understood superficially. Although some students could, most of the students are not able to carry through a project from question to data collection to analysis, to make decisions about constructing a graph. Most students did not seek out the source on a regular basis unless they were the source of information.

They did learn to ask more questions or at least to be more suspicious of information they read or hear on TV. One example occurred when they didn't want to sign the permission form to be interviewed for the RABEM project. They questioned profusely what the information was going to be used for even though they had been part of the formative process and knew Martha and Bonnie.

When given the information from a question most are able to construct bar graphs. When constructing graphs, they can plot points, have an x and y axis, label increments, title, read a table, and interpret circle graphs.

CONCLUSIONS

Keeping Our Students Alert and Present.

An important point to keep in mind when planning our lessons is within a three-hour period most students' attention was focused in the beginning. If new concepts were to be addressed, they needed to be introduced early in the class period. We learned that having food was one way to keep them in the present. Their minds stayed with their tastebuds instead of wandering home to deal with personal problems or hunger.

Learning A New Term Took Many Lessons.

We emphasized the concept of data during each class. They seemed to understand it the first time it was reviewed so we planned to build upon it. It became apparent when they had not retained concepts from one class period to another. It took most students many lessons to grasp it. Mabel, for example, kept using the same examples (data processing, data base) but could not create a working definition.

We believe this has a larger implication for the ABE field. The tendency is to create self-contained lessons because of poor attendance. Teachers cannot rely on the same students being present so they avoid planning continuous lessons over time. We found that at the end of a class, a student understood the lesson of the day, but we could not guarantee that they would still remember the concept the following week.

Memorable Lessons Created A Strong Foundation.

Because this math class met only one time per week, having a lesson that people would remember was crucial. We would refer to the four lessons described with a word or sentence and tap what was learned in them. This provided more examples for the students to bring up which enriched their understanding.

Least Educated, Lowest Level Readers Could Excel at Reasoning, Questioning and Made The Biggest (Most Important) Connections.

Too often the assumption is that a low level reader and math student must master computation before beginning higher level thinking skills. We anticipated frustration from the most beginning level. We anticipated having to slow down for them, having to build in success. We didn't anticipate that Ethel would demonstrate such aptitude in reasoning and making connections. We will continue expanding the math activities we do in class because we know that the beginners can keep up, indeed can lead the class.

Accommodating Learning Styles.

Our last finding/conclusion is that within the scope of a unit on statistics we had to be flexible enough to accommodate the various learning styles. We needed to show the whole process for students like Robert. We needed to draw actual graphs for students like Mabel and Anna. We needed to come up with our own data for students like Ethel. We needed to recognize individual strengths like Anna's estimation talent. And we need to go back and forth between the familiar and the unfamiliar: raisins and bagels, between numbers of raisins per box and flavor of bagels sold, between race as a category and country of origin as a category.

NOTES

[1] Darrell Huff, *How to Lie with Statistics*, (New York: W.W. Norton & Co., Ltd., 1982) p. 3

[2] *Boston Globe*, March 22, 1993

[3] Susan N. Friel, Janice R. Mokrus, and Susan Jo Russell, *Statistics: Middles, Means, and In-Betweens* from *Used Numbers: Real Data In The Classroom*, (California: Dale Seymour Publications, 1992) pp. 13-20

[4] Poll, "If you could fill a bag full of bagels, what kind would you put inside? Marble? Onion? Plain? Raisin? We polled 172 kids and teachers to find out what their favorite bagel is." *Boston Globe*

“WHAT IF I USED RESEARCH AND THE SCIENTIFIC METHOD TO TEACH PROBLEM SOLVING?”

By Catherine M. Coleman, Worcester Adult Learning Center

The question that I chose as the focus of my research is “What would happen if I used student research and the scientific method to teach problem solving skills?” This particular question intrigued me for a number of reasons. I believe that the scientific method of questioning, observing testing and generating conclusions and/or new questions can be broadly applied to problem solving in practical, “real life” problems as well as to the type of math word problems that adult learners see on the G.E.D. test. Most of us don't see ourselves forming hypotheses in order to solve problems; nevertheless, we do this every day. We look at the facts of the situation, make an educated guess about what is happening in the situation, and test our ideas in a variety of ways to see if we were right.

> ***“Most adult basic education learners are developmentally able to reason abstractly, but many have had little opportunity to practice formalized reasoning while doing math...***
>
> ***An atmosphere must be established where learners are afforded many opportunities to explore, to apply reasoning skills, to ask questions and to examine and validate their own thinking.”***
>
> MASSACHUSETTS ADULT BASIC EDUCATION MATH STANDARDS, STANDARD 3: MATHEMATICS AS REASONING

One reason I chose this question is my desire to build in my students the ability and confidence to question. The first and most basic step of the scientific method is questioning. Indeed, our existing body of scientific knowledge began when someone had the courage to ask “Why” or “How” or “What would happen if...” The confidence to question and to take risks is, in my opinion, a necessary prerequisite for effective problem solving. I have often seen adult learners who lack this confidence. Many of the learners I have worked with over the years have come to view their education not as an activity in which they are the key participants, but as a spectator sport in which they watch and listen as others “play the game,” so to speak. They often listen to the voices of authority (teachers, textbooks, media, and government) and drown out their own voices, and in so doing come to know things not through their past experiences and reasoning process but through others'.

Many adult learners with whom I have worked have made comments which echoed these ideas. “You tell me.

You're the teacher!" "How could this answer be wrong? The book says it's right." Many times there will be a surprised blank look when I ask questions like, "You solved the problem differently than I did, but we both got the same answer. Can you explain to me how you did it?" Often, when that happens, my students immediately assume that they got the right answer but "did it the wrong way," and seem surprised when I tell them I want to know more about their thinking about the problem. Their comments seem to underlie a sense of low self-esteem and powerlessness. They don't seem to feel they have the right to question the authority (in this case me) about anything. I am interested in learning whether the use of the scientific method of inquiry will increase their ability to question the "authority," whether that authority is I or other people or agencies in their lives.

Another reason for choosing this question is the concept of understanding research. Research is something that we come into contact with many times in the course of everyday life. Advertisers tell us that "9 out of 10 people surveyed chose Crest as their toothpaste." The medical profession tells us that "research indicates that a high cholesterol level can lead to heart failure." A parent of a child in danger of staying back is told that research shows that "75% of children who repeat a grade benefit from the experience."

How do the adult learners that we work with interpret these statements? While everyone is different, it has been my experience that many of the learners that I have worked with have not interpreted these statements at all but accepted them as facts without understanding how they came about or questioning their validity.

In the example of the research regarding children who stay back, what if they only studied four people? What if all the people who answered questions about Crest worked for the company who makes Crest? How can medical research tell you one day to stop eating butter and eat margarine, and then tell you next to stop eating margarine because it is just as bad as butter? Without a basic understanding of the scientific method and research and without the confidence to question, I am afraid many of my learners will simply accept things as facts which may, in fact, hold no truth at all.

My choice of the question, "What would happen if I used student research and the scientific method to teach problem solving skills?" is based on my hope that experience with generating their own questions and learning about the process of finding the answers to those questions will help my adult learners to look with the eyes of a scientist at the world around them, to explore ideas, to feel confident in questioning, to reason and to use the scientific method of inquiry as a tool in problem solving.

The remainder of this paper will be devoted to presenting my research on that subject. I will first describe the context in which the research took place. I will describe the important factors in our learning environment which I believe had a significant impact on the results of the research. The research had three stages: an initial assessment, the student research piece, and the application to traditional problem solving. Within the student research piece, I describe two scenarios: the raisin experiment and our work on anxiety. I will then discuss the results that were obtained, and the conclusions and implications for further research which need to be addressed.

THE CONTEXT

My research was conducted in my adult basic education class at the Worcester Adult Learning Center in Worcester, Massachusetts, an urban adult education center. Frequently the challenges that surface in this class are traceable to the scattered levels of the students and to interruptions particularly from mandated testing.

My class consists of seven women whose English proficiency, reading, writing, and math ability are as diverse as their ethnicity and first language. They range in age from 27 to 45. Spanish is the first language of nearly half of my class (three students). One Latina learned English first but then moved back and forth from the United States to Puerto Rico and forgot much of her English. Greek is the first language of one student and English is the first language of the two born and raised in Worcester. Of the five students for whom English is their second language, four were assessed as needing no ESL based on the results of the BEST test (Basic English Skills Test). Their levels of English proficiency (as assessed by their oral and written communication) vary greatly. All five try hard but struggle with English language a great deal.

All of the students in the class are between the 0-5.9 grade level range as assessed by the TABE (Test for Adult Basic Education). Most teachers find meeting the needs of students with this broad a range of scores to be difficult. Yet these scores belie an even greater discrepancy in levels. The grade level scores in the three areas are averaged to get the composite score on which their placement is based. Students vary greatly however, within the broad range of 0-5.9. Some students have reading comprehension levels in the 9-10 grade level while having language and math scores in the 3-4 grade level. Other students have composite reading levels in the 2-3 grade level (possibly due to language-related problems with the English vocabulary) and math levels in the 6-8 range.

We certainly are a very mixed bunch! This has proven to be both an asset and a liability. While it is difficult to plan activities which would be appropriate for all levels, the variation in levels has helped to promote some cooperative learning as students have (on their own) paired up and worked on various problems. The variation in ethnic background has been very valuable and interesting as we all learn about our various countries and cultures.

Questioning, exploring, and thinking require time and a trusting and safe environment. It is difficult to feel safe in your environment when the rug could be pulled out from under you at any time. Unfortunately, this is sometimes the case. Interruptions and mandatory testing (as per Welfare requirements) often play a role in creating anxiety and a lack of continuity for the class.

This year in particular, we experienced many interruptions. Due to circumstances beyond our control, our class had to move three times this year. We also missed an unusually high number classes this year due to snow days.

In addition to this, five of my seven students are Welfare clients and are mandatorily assessed by Welfare using the TABE every 180-200 hours of class time. They are expected to make significant improvement (a grade level or higher) and are sometimes required to change to a full-time educational program (25 hours class time or over) or to end their educational program and look for work if they do not make the required amount of progress. As a result of this assessment procedure, a lot of material must be covered in a short amount of time. It has been my experience that this kind of pressure can be counterproductive to a positive learning environment. While accountability is necessary, it is unfortunate when this kind of accountability occurs at the expense of a more in-depth, valuable, lasting learning experience.

METHODS AND FINDINGS

What would happen if I used student research and the scientific method to teach problem solving skills? To answer this question, I needed to first teach the scientific method and engage my students in research. Although this approach was my idea, I wanted my students to determine the topics for our endeavor. In order to get their input and gather information that would help me address their varied levels, I created a questionnaire and conducted an in-class survey about math and science. The results of this survey were the starting point for my own learning. The first conclusion I came to based on the survey was that the class seemed to have a fairly limited idea of what math was all about (numbers). None of the students who completed the questionnaire connected math to problem solving or to anything in "real life."

More people than I would have expected had background in science. Since so many of the learners I have worked with had experienced great difficulty during their secondary grades, I expected that most of my students probably wouldn't have much background in science or remember about it. I was wrong. People seemed to have quite vivid memories of their science classes and seemed to have a significant interest in it. It's amazing what you find out when you ask the right questions!

Sample student-generated questions

"How will mental abuse affect a person?"

"What makes people kill themselves?"

"I want to know about anxiety. What causes it? What can I do about it?"

"I want to learn about the brain. How can I memorize things?"

"Why do I feel depressed?"

"How does alcohol and drugs affect other people?"

"What is anxiety?"

Compiling the results of the survey was a good experience for the class. Among the big winners of "science topics of interest" were memory and cancer, but the biggest winner of all was a write-in vote—psychology. In order to narrow down our topic, we then did some discussing and brainstorming about the very broad subject of psychology. We compiled a class list on the board of about thirty subtopics. I then asked them to write their own questions about any of those subtopics that interested them the most. The questions I received were as varied as the class. However, six questions were related to the topic of anxiety, and so anxiety became our topic.

Another thing that I found interesting was the ease with which the class generated questions regarding psychology. I only asked them to write three questions, but I don't think they would have had any difficulty coming up with many more. It seems that, if given the chance, my adult learners could let their curiosity lead them and become more active participants in the learning process.

The next step was to introduce the idea of the scientific method. I did this by asking what they would do at home if the lights suddenly went out. I got this idea from some reading I found in the SABES (System for Adult Basic Education Support) Science Box. Many people in the class initially responded that they would call the electric company. Little by little though, some other ideas came up. One person said, "Well, I'd make sure it wasn't just a light bulb that went out or something." Then other people asked questions such as "Did just my apartment's lights go out or was it the whole building?: After we had discussed these ideas for a while, I introduced the term "scientific method" along with the five steps involved in it. One conclusion everyone seemed to draw that day was that no one would ever be able to say or spell the word "hypothesis" correctly. I told them that the idea was more important than the spelling or the pronunciation. This is how we began out discussion of the scientific method.

I taped this discussion, and as I listened to the tape, I was struck by how much silence there was on their part and how much talking and questioning there was on my part. People seemed somewhat overwhelmed by the terminology. The response, "Call the electric company" was not only their first, but seemed, even at the end of the discussion, to be their option of choice. For the most part, they seemed willing to leave it at that, and I wonder if the questions they arrived at were for my benefit alone.

My first thought was that this confirmed my belief that they were "swallowing things whole," and that they were depending on the authority (the electric company)) to solve the problem without doing any further investigation on their own. However, I think it is also important to consider the possibility that this was a reflection of a healthy respect of electricity and/or lack of background regarding electricity. Perhaps it was a little of both. In any case, I knew that I wanted my next activity to be more hands on and to give them the opportunity to see the scientific method in action.

A few weeks later, we did another class activity to illustrate the principals of the scientific method. We did an experiment with raisins. I found this experiment in the Dale Seymour book, *Used Numbers: Real Data in the Classroom (Statistics: The Shape of the Data)*. In this activity, the idea of statistics is introduced and the concept of the scientific method is reinforced through an experiment. For this experiment, I brought in eight boxes of 1-1/2 oz. boxes of raisins (all Sunmaid brand). I gave each student a box and asked them this question, "How many raisins do you think are in the box you are holding in your hand?:" I told everyone that they could do anything they wanted to answer the question except open the box. Students began to look at and shake the box in different ways. After a few minutes, everyone was asked to write down an estimate of how many raisins they thought were in the box.

It was very interesting to watch everyone as they tried to make their estimate. I wasn't sure what to expect. Based on our discussion about the electricity going out, I wondered if some of the class mighty just take a guess without much investigation. Only one person seemed to do this though.

The estimates varied greatly, going from 50-150. Everyone then tested their hypothesis by counting how many actual raisin;s were in the box. Looking back at the pictures and videotape from this class, I noted the level of enjoyment and the comments regarding the way that various people made their estimates. For example, about how big each raisin would be.

We then listed our data on the board and made some observations about it. The first way everyone wanted to look at the data was in terms of how close everyone was to the real number of raisins in the box. Our analysis revealed some students were close (within 4-5 raisins); some were way off (70-100 raisins over); no one guessed less than there actually were. We looked at the range of the estimates and the range of the actual number of raisins, and we talked about how and why the range of the estimates was 150, and the range of the actual number of raisins was much smaller (9 raisins). The concept and term "range" was a new one for the class.

A very interesting discussion followed about what the possible reasons were for why one person had 81 raisins and one person had 90 raisins. We discussed the concept that many people in the class had previously seen on food labels "packed by weight not by volume." After shaking the boxes, everyone agreed that their boxes of raisins weighed about the same, so how could some people have more raisins than others? Maritza suggested that maybe some raisins were bigger than others. She compared some of the raisins and proved this to be true. Some students seemed to be somewhat embarrassed that their estimates were so far off track. We talked about why they thought that their estimate were so high. Carmen said that sometimes the raisins stick together, and then they make it feel heavy. This could make you think that there were more raisins than there really were.

I was impressed with some of their explanations. People were very interested in finding out why their estimates were so different. The energy level in the room during this part of the discussion was noticeably high.

Then we began to discuss the concept of finding the middle of our data. We looked at the data and students were asked to tell me what number would be about the middle of the data. For the actual number, people said "about 80." We talked about the idea of finding an averaged number of raisins in the 1-1/2 oz. box. I asked why 90 wouldn't be the average number of raisins in a box, and at first, the class seemed very confused until one person said that only one person had 90. I asked if she thought that this was a "normal" or "unusual" box of raisins. They agreed that it wasn't normal because most people had about 80 not about 90. We then went over the way to find an arithmetic average and found one for both the estimate data and the actual data.

At a later date, I had the class represent their data as a whole group. They created a group bar graph which showed the estimates and the actual data. Also on the same date, I extended the activity by bringing in two different brands of raisins (Dole and Finast, the same 1-1/2 oz. size) and asked if people thought the number of raisins would be the same or different. They made estimates and then this time were allowed to revise their estimates based on looking at the tip layer of raisins in the box. After seeing the top layer, a lot of students revised their estimates to lower numbers based on the fact that these raisins seemed to be much bigger than the other brand's raisins Maritza noted that these raisins seemed much fresher than the others. I asked her how she knew that, and she stated that they looked fatter. Another student, Carmen, said that if they were fresh, they would be "wetter" (she struggled to find the right English word and eventually said " moist"). The other brand raisins did stick together. They were dry.

As we discussed these things, we talked about the steps of the scientific method and how scientists sometimes come up with new question in the course of their investigations as we did.

As luck would have it, I happened to have a 15 oz. box of Sunmaid raisins in my cupboard. I decided to bring the box to class partly as a joke and partly to see whether the class could make estimates based on their experience with the 1.5 oz. box.

The whole group laughed when I brought in the big box of raisins, and said, "We don't have to count all those raisins do we?" The class did make educated guesses as to how many raisins were in that box, but since no one wanted to count all those raisins, decided not to test that hypothesis.

Although the steps of the scientific method were discussed at various points during the raisin experiment and the extension f the experiment, I did not get the feeling that the class really "got it." They seemed to enjoy the experiment with the raisins, but still did not seem particularly clear about the idea of estimation, even though we had done quite a bit with estimation in math prior to this. I wasn't sure they were connecting the idea of estimating with the concept of hypothesis. I needed to try it a different way. The students needed practice forming and testing hypotheses.

I was also concerned that this wasn't directly related to the question that the class had chosen, so I spent some time looking at their different questions that the class had raised about anxiety: "What can I do about anxiety?" "What are the symptoms of anxiety?" "What is the connection between the mind and the body?" "Why do I always eat when I feel nervous?" "Why can't I eat when I get anxious?" "How can I relax?" The group had asked so many great questions. I worried that not many of them would be easily tested though. I guess you might say I was getting a little "anxious" about it. Many of the students had asked the question, "What is anxiety?" I began thinking of ways that we could measure anxiety and define it. Finding out about the Boston Science Museum's Human Body Discovery Space and meeting staff members Mary Dussault and Lucy Kirshner alleviated my anxiety. I arranged for a class trip to the museum.

During the week prior to the museum trip, I asked everyone to write one question and one hypothesis regarding anxiety. I told them that we would test them if possible when we made our trip to the museum. The questions mostly revolved around reactions to anxiety and were very similar to the questions they had asked when we started talking about science and did our survey several months earlier. Writing the questions seemed easy for them, but writing a hypothesis was more difficult, at least in terms of writing ones that were testable. Here are some examples of the class's questions and hypotheses:

Question: How can I control my anxiety?

Hypotheses: Try not to think about the problems.

Question: Why when I have anxiety, do I want to clean and clean?

Hypotheses: Because I feel alone, and I don't know what I am doing.

Question: What is anxiety? Could it be because you're nervous?

Hypothesis: It is caused by nerves.

Question: Why when I have anxiety, I eat more?

Hypothesis: I think because my mind runs faster.

This exercise helped students take their initial questions one step further. We spent some time talking about testability and what makes a hypothesis testable. I found myself offering guidance when we had to choose a hypothesis that could inform a testable experiment. This step was a consistent stumbling block. They seemed very confused about this issue. After some discussion, we decided to choose Carmen's Question ("How can I control my anxiety?") as a focus for our experiment at the science museum.

Hypothesis in hand, we were ready to go to the museum. After museum volunteers explained how a biofeedback machine defines and measures anxiety, students had an opportunity to measure their own anxiety levels. We did various tests to both raise and lower each others' anxiety levels. For example, I tried to startle people by shouting. For some people, I asked a complicated math question. For others, I reminded them that the GED was a difficult test. The whole group was able to observe as well as participate in the testing of each other's hypothesis.

Later on, when we got back to class, we finished our experiment by completing the steps for the scientific method: analyzing our data (which we did by graphing individual results and comparing them) and discussing our results in order to form conclusions and new questions. We had a lot of discussion of the differences among the members of the class in terms of what made some people anxious and what made other people anxious. With Jane, all I had to do was mention the word "GED" and her anxiety level as measured on the graph was off the scale! Maritza's anxiety level went up as she tried to spell difficult words. The shape of some student's data was steady, while others' data were erratic.

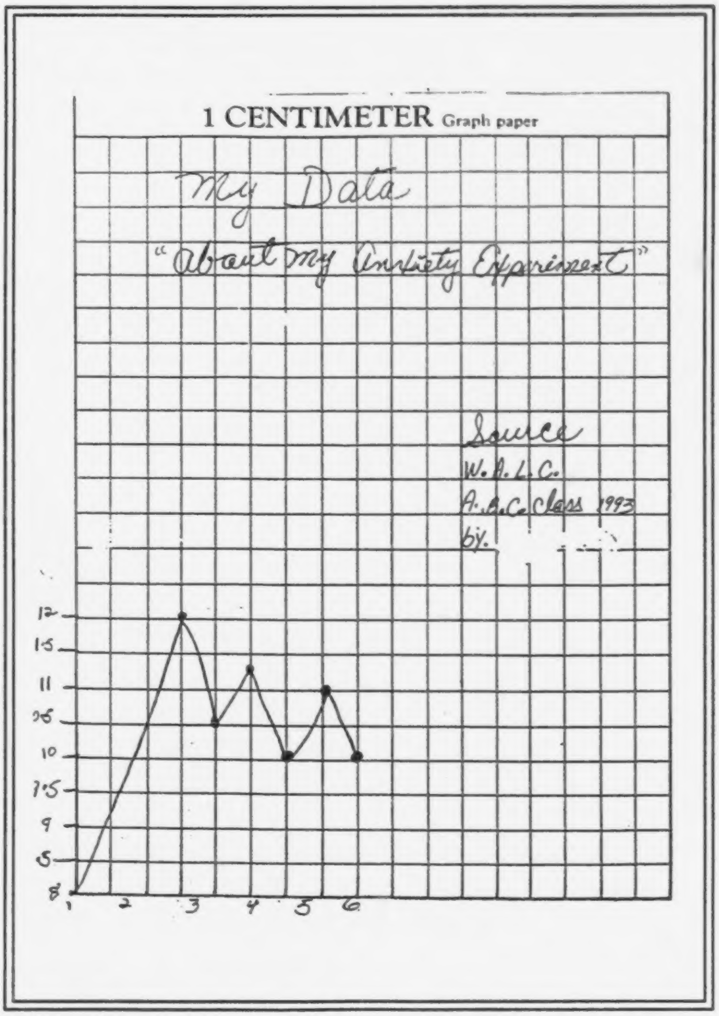
The Biofeedback Machine

The Biofeedback Machine:

- measures the conductivity of a person's skin
- uses units called micro mhos
- prints a graph
- sends electrical current through the index finger
- is sensitive to sweat

Our main question focused on finding a way to control their anxiety. Almost everyone's level went down after using a deep breathing exercise. A shoulder massage given by one student helped one person's anxiety level go down, while a shoulder massage given by me to this same person sent her level right back up. This raised some fascinating questions about the meaning of the same action performed by different people.

Having a focused and testable question with the actual equipment available to do the experiment was a breakthrough. We proved that individuals can control their anxiety using their minds and that those around us can have a negative impact on our anxiety levels. In true scientific fashion, we generated new questions while we continued to pursue research in class. I was ready to move on to the next part of my own research question: How will the scientific method apply to problem solving? I wanted to see how an understanding of the scientific method would help or transfer when students confront a traditional math problem.



APPLICATIONS TO PROBLEM SOLVING

To test this hypothesis, I chose two word problems and created one "real life" problem which I gave to four of my students. I did a think-aloud protocol with each of the four students on each of the three problems. I also interviewed each of the four students in order to gain insight into their experiences this year with math and science. I wanted to find out what they understood about research and the scientific method in order to see if they would apply what they had learned to three different problems.

The problems I chose for these interviews (the "Day at the Beach" and the "Eggs" problem) were chosen

What is the scientifi method?

The scientifi method is good because help me to understand easy like asking or meke a Hypathesis and meke a Data. I like when we all work in the group and make the Raisen Data graf it was so easy to learn.

Carmen

For me it was a whole new experiance one that you usually don't have a chance to work with until yore in collage. I was very glad to be able to have the chance to experiment with the brain in this class.

Carolyn

based on their suitability for use with the scientific method of inquiry. Both problems are multistep, non-routine problems which encourage learners to use a variety of problem-solving and reasoning strategies. They both encourage a person to make reasonable inferences and form hypotheses based on the facts that are given in the problem. In both problems, there is a certain amount of trial and error that is likely to occur, thereby providing an opportunity to experiment with the various possible solutions as one tries to solve the problems. Both require a certain amount of number sense in terms of knowing which numbers are possible solutions and which numbers are not.

A DAY AT THE BEACH

Esther's first purchase on her trip to the beach was 2 glasses of lemonade and a popsicle. She paid 80 cents for them all. Later the same day she bought 3 glasses of lemonade and 2 popsicles for $1.30. How much does 1 glass of lemonade cost?

1. .20
2. .30
3. .50
4. 1.30
5. Not enough information is given.

THE "EGGS" PROBLEM

There are fewer than six dozen eggs in a basket. If I count them two at a time, there is one left over. If I count them three at a time, there are none left over. If I count them 4,5,or 6 at a time, there are always three left over. How many eggs are there?

The third problem was one I created out of my own experience and was designed to gain insight into my learners' ability to use the scientific method in "real life." The problem was as follows: "I currently have a car which has air conditioning. Since I have never owned a car with air conditioning before, I am interested in finding out: Does my car use more gas when I use the air conditioning?" For this paper, though, I've chosen to focus on the egg problem.

All four students had considerable difficulty with the "egg" problem. Out of four students, only one has solved the problem to date. (However, the other three tell me they are still trying and we have been out of school for two weeks!) This is an interesting finding in itself. All four students first multiplied 12 x 6 to get 72 eggs. Most seemed a little lost after this point, however. Some initially thought 72 must be the correct answer and tried the various "tests" to see. They tried to count them by 2,3,4,5, and 6. Others concluded that "fewer" than 6 dozen eggs" that must mean 5 dozen, 4 dozen, etc.; and therefore, started out with numbers like 60 or 48. One student, Carolyn, read the sentence "If I count them 2 at a time, there is one left over" as "If I counted them two at a time is there one left over?"

Different people used different methods to test their solutions. One student, Carolyn, employed the strategy of using tally marks to count by two or three using paper and pencil. Carmen and Jane surmised after trying 2 or 3 numbers that it must be an odd number because "if I count by two, there is one left over." Carmen counted by 5 aloud in spanish. Everyone was able to count by 2 or 5; no one counted by 3, 4 or 6, though, without using pen and paper division or tally marks.

Maritza solved the egg problem much to her and my delight. She told me she figured it out after trying many different numbers and after using "that thing you gave me last week". The "thing" to which she was referring was an activity from a book called *Those Amazing Tables*. It has a typical multiplication chart in which the student is supposed to highlight various numbers and patterns. The goal is for the student to see the patterns inherent in the times tables. It took her several hours of hard work, and we were both thrilled with her results.

The egg problem clearly illustrated students using the scientific method to solve the problem. First, they needed to understand the question. Everyone took a while (and a few times reading it over) to do this. Then they began hypothesizing various solutions and testing those hypotheses using the information in the problem. Although on one said, "Now I am making my hypotheses," they made them and tested them nonetheless.

In addition to the problems I asked the students to see if they saw any connection between the scientific method and the problems that she had just done (lemonade, egg, and air condition). She said, "They all have a lot of numbers and a lot of thinking." "When I get the problem, I have to figure it out and do a process." When I asked Carolyn the same question, she replied, "The order of things, the sequence." I asked her to tell me more about what she meant by that and she went on, "I have a hard time picturing the problem in my mind. I have to separate the words from the numbers. I have to decide the most important facts."

I then asked the students to tell me if they thought that the scientific method could be used outside of school. To this question, Carmen replied, "Yes, like in the raisins. When we go shopping, we compare—bigger or smaller and the price. One price may be less and we get less food. We cannot open the package to see how much is in it. We must make an estimate and then see later if it is right." Maritza said, "Like the air conditioner. You have to do tests." Carolyn said, "You can use it in science experiments like scientists." I asked if she thought the average non-scientist could use it. She said she didn't think so. Jane also wasn't sure how it could be used outside of school.

"What would happen if I used student research and the scientific method to teach problem solving skills?" As I spoke with my learners, read their writings, and listened in on their thinking processes, I found that several things happened. The first being that the class seemed to become more willing and able to communicate their thinking. When I asked, all were willing (and some even seemed eager) to participate in the research interview as well as to try some new ways of learning math and science. Although during the interviews, as well as during class, people seemed to have a hard time verbalizing their thoughts, everyone seemed eager to try to explain what they were doing to solve problems. (One student told me during the interview, "It's hard to explain what I am doing and doing it at the same time.")

The class was able to generate their own questions and seemed to do so with more frequency than they did previous to this research. They had many wonderful questions about psychology and about anxiety which they wrote. There were also many questions at the Boston Science Museum which were asked but not written, such as "Why are her reflexes faster than mine?" "How come I can squeeze harder with my left hand even though I am right handed?"

Although only one out of four students interviewed was able to solve the egg problem, with the exception of one student, no one seemed so overwhelmed and discouraged as to not try the problem. All were able to choose numbers based on the facts within the problem and to make new choices based on the results of their own mathematical experiences/experiments with them. I was surprised to see how much they all seemed to actually enjoy the challenge of the situation. All four of the students interviewed brought the egg and lemonade problems home to their families and did further work on it on their own time! They also did not seem hesitant to question me about the problem in the ensuing weeks.

Certain students in the class seemed to come alive this year. Maritza, who a year ago didn't say much at all, became an active participant in the projects and experiments we did during this research. She was able to communicate her thinking to the other members of the group and help them to understand problems in a different way. Carmen told me of her experiences at the grocery store when she tried to compare prices and products and see which ones were better. Maritza and Carmen exercise their ability to question more now than they had previous to this year. I hope that continues past this year. I think that it will.

CONCLUSIONS

As my class learned this year, good research provides not only answers but also more questions. As I end this project, I am thinking of questions which came up during this project that I would like to investigate in the future:

- ✔ What other types of activities could promote the assimilation of the scientific method into my students' repertoire of problem-solving techniques?
- ✔ Would learning the scientific method and basic research methods help adult education students to understand mean, median, fractions, decimals, and percents?
- ✔ What effect does learning the scientific method have on standardized test scores such as the TABE and the GED?

✔ What would happen if I found a way to connect the scientific method to the worlds of business and industry?

This project has been a learning experience not just for my students but for me as well. I have seen the excitement and personal involvement my students seem to have when activities are hands-on and related to their lives. I have seen growth not only in my students, but also in myself. I have changed my practice quite a bit. I have tried to incorporate small group, cooperative learning into more lessons, and I have begun to use manipulatives in my math and science lessons. I have tried to start from where my students are, not where I'd like them to be. Doing the interviews and the think-aloud protocols with them, enabled me to understand much more about their thinking and where they go off the track.

To me, the NCTM *Standards* represent a philosophy of education—a philosophy which encourages independent thinking and inquiry, which challenges students to make conjectures to validate their thinking and to effectively communicate their ideas. As I think about how this applies to my adult learners, one word comes to mind—empowerment. My implementation of the standards using student research... The scientific method gave my students the opportunity to more fully explore the power of their own minds and to value the diversity of approaches inherent in any group. Based on the students' comments, both verbal and written, and the results of my interviews with them, I think the seeds for further growth have been planted. As Maritza said when I confirmed that her solution to the egg problem was correct, "It is good to try new things."

MATHEMATICS AS COMMUNICATION

JOURNEY INTO JOURNAL JOTTINGS

By Donna Curry, Digital Equipment Corporation

For the past three years, I have worked closely with a teacher who has been extensively trained in the whole language approach. She is very knowledgeable and innovative in curriculum areas involving language. However, she is a "math phobic", having received very little math training even though she is certified in elementary education. In one of our many discussions about this lack of math background, I questioned why she had such vast exposure to whole language yet so little in math. She explained that she was mentored by a whole language "guru" who believed that the whole language process could be applied in any area of the curriculum — any area EXCEPT math.

> ***"In the adult basic education classroom, curriculum design must include approaches to teaching mathematics as communication which allow the learner to discuss with others, reflect and clarify their own thinking about mathematical outcomes."***
>
> MASSACHUSETTS ADULT BASIC EDUCATION MATH STANDARDS, STANDARD 2: MATHEMATICS AS COMMUNICATION

This bothered me tremendously. I believe that ALL areas of the curriculum can and should be integrated, and that language flows throughout everything we learn and do. This belief, and the assumption by so-called educational experts that math should be treated differently from other curriculum areas, spurred me to design a research project around the use of language in my math class. I believe that "communicating" in math is an important step in learning math, so I decided to look more closely at Massachusetts Adult Basic Education Math Standard 2: Mathematics as Communication, and specifically at journal writing.

I asked myself:

"Do students gain a better understanding of and broader insight into math concepts if they write about what they are learning?"

PLAN OF ACTION

The research project, to be implemented in the context of an actual workplace classroom environment, consisted of three sections: journal writing, written assessments, and one-on-one interviews. I will elaborate on the process of each of these in greater detail.

I envisioned the journal writing segment being an integral part of each class period. The journal writing was to be a combination learning log and response journal. Students would write about what they were learning in math and I would respond to their jottings.

I intended to interview students both midway through the class and at the end of the nine weeks of classes. I planned to give the assessment at the beginning and at the end of the cycle. I wanted to obtain both "hard" and "soft" data and designed the assessment accordingly.

DESCRIPTION OF CLASS

The class, Prep Math, is the final class in a series of four courses designed to bring employees from a very limited understanding of math up to a level at which they would have adequate math skills to successfully pass the math portion of the Certified Quality Technician (CQT) test.

The CQT exam is a nationally recognized standardized test used by the American Society of Quality Control (ASQC) to recognize individuals as having a set of basic knowledge about quality concepts. The company in which I was teaching encouraged employees to become certified as quality technicians; the national recognition would lend credibility to the individual as well as to the company.

As is typical of workplace education classes, this class was driven by several agendas.

As is typical of workplace education classes, this class was driven by several agendas: the company was looking for employees who could become nationally certified; employees had their own agendas which often revolved around their becoming better skilled to maintain their position within the company. As instructor, I had my own agenda. I wanted students to become more comfortable with math, more willing to learn and attempt new strategies. I wanted them to become comfortable tackling any kind of math problem. I also wanted them to be able to apply what was being taught not only during the CQT test but also on the job and in their daily lives. Lastly, I wanted them to enjoy the experience of learning math.

Employees who participated in the Prep Math class had either completed the other three math classes or had successfully passed a teacher-made assessment. Employees participating in Prep Math had already had some exposure to fractions/decimals/percents, problem-solving strategies, algebraic notation, and Pareto charts and histograms.

Prep Math covered the following topics:

✔ Notation (algebraic and scientific)

✔ Metric system (including metric and SI conversions)

✔ Geometry (perimeter, area, volume)

✔ Probability (including a brief introduction to permutations and combinations since sampling in the workplace is based on these concepts)

✔ Histograms, measures of central tendencies (mean, median, mode), and measures of dispersion (range and an introduction to standard deviation).

PROCESS OF THE PROJECT

Journal/Learning Log Writing

The students were originally asked to respond to three questions during the last ten minutes of class each day:

1. **What "struck" you?** Was there anything during the session that made you say, "Aha"?
2. **What didn't you understand about today's lesson?** Was there anything that needed further clarification, more examples, more time?
3. **Did you learn anything new today?** Or did you remember something that had been long forgotten?

Additionally, because this was a response journal, I asked the students to respond to my questions. My desire was to encourage students to begin to use the language of math, as well as to write more extensively. I responded to them on a daily basis; therefore, they had to answer an extra question or two beyond those listed above.

I stated at the onset of the course that all journal writing entries would be confidential, although I did share with them that this was a research project and I received their permission to use their writings in the context of my research "story". Students were free to write about whatever they felt compelled to, but I did try to guide them with the questions above.

I also explained to the students up front that I didn't care about the mechanics of writing. I wanted them to express on paper what they were learning and what issues they had. During the entire process of journal writing, no student ever asked me for help in writing, nor did anyone feel compelled to resort to the dictionary.

Sometime during the research project, one of my peers suggested I ask students about a topic BEFORE it was formally introduced. She thought this strategy might prove useful in allowing students to see what they learned in a very short time period. The class session before introducing the topic of "Probability", I tried this strategy. It was difficult to determine if this strategy was effective as events in the workplace (lay-offs) impacted our group just as the topic was introduced. I would like to try this strategy again, because I think it could be a vehicle for ongoing assessment.

Interviews

I scheduled interviews at two different times: halfway through the cycle and at the conclusion of the entire seventeen sessions. My goal was to interview each student on a one-on-one basis. Because of time constraints, the interviews scheduled for midway through the course never transpired. I did succeed, however, in interviewing the students at the end of the session.

The purpose of the interviews was to elicit input about the value (or lack) of journal writing in the context of a math course. I tried to structure the questions so students would feel free to offer what might be construed as "negative" feedback. I wanted students to be completely honest with me. I was concerned that, because I knew they respected me, they might be reluctant to give me any "bad news" about the class. I came up with these questions for the one-on-one exit interviews:

What did you think about having to write in a journal during math class?

What were the positive and/or negative aspects of journal writing?

Would you suggest journal writing be included in another math class or could the time have been better spent? Why?

Do you ever intend to write in a journal again? Why?

In addition to the questions above, I asked two more. These last two questions digressed from the discussion of journals, but I wanted to know this information for myself as instructor. I also wondered if the information would lead me to discover any other connections.

What was the most significant learning during the math cycle?

If you could change the class, what change(s) would you bring about, and why?

Pre/Post Assessment

I designed an assessment that could be used both at the beginning and at the conclusion of the course. I framed the questions to gain insight into how the math class had impacted the students. I hoped to see an change in three areas:

1. **Attitude** - By participating in the math class, and specifically by writing about what they were learning, would students' attitude toward math change?
2. **Behavior** - Would the math class have an impact on what students DO regarding math? Would students be more willing to participate in activities involving math? Would they feel more prepared to take responsibility for math-related tasks?

3. **Knowledge** - Although it would be reassuring to learn that students had a more positive attitude toward and were willing to use math, I also wanted to know if they had LEARNED anything new in math. Had the students' knowledge base increased since participating in the math course?

(These three areas are keyed in on the math assessment in the appendix.)

Because I wanted both statistical data and anecdotal information, the assessment contained two types of statements. The first twelve statements involved a rating scale of 1-10, 1 being at one extreme, and 10 being at the other extreme. I chose a range of 1-10 to provide a wider degree of options for students to choose from.

The second part of the assessment consisted of nine open-ended statements. These were also designed to provide me with feedback on the same three areas: attitude, behavior, and knowledge.

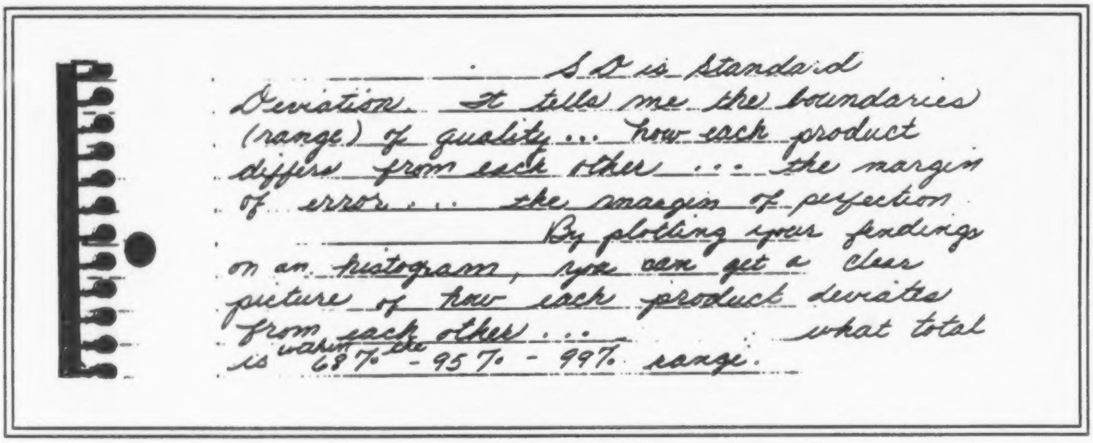
The Language of Math

CONTENT OF JOURNAL

Students may not have written as much as I would have liked (or expected), but they all wrote. They all expressed their views about learning math. I did find that some of the students became more willing to write as the weeks progressed. The quality of their writing did not necessarily improve, but the quantity definitely did. I was pleased to see that they began to use the language of math with more regularity and were even simplifying the vocabulary to suit their needs.

For example, this is the first time I have seen standard deviation defined without having to refer to the unwieldy formula!

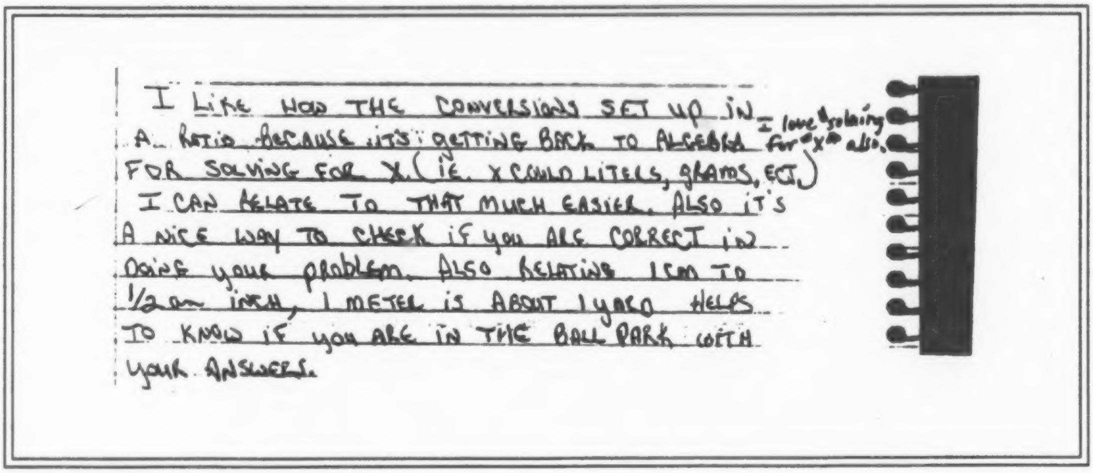
(see example, next page)



S.D. is Standard Deviation. It tells me the boundaries (range) of quality ... how each product differs from each other ... the margin of error ... the margin of perfection

By plotting your findings on an histogram, you can get a clear picture of how each product deviates from each other ... what total is within the 68% - 95% - 99% range.

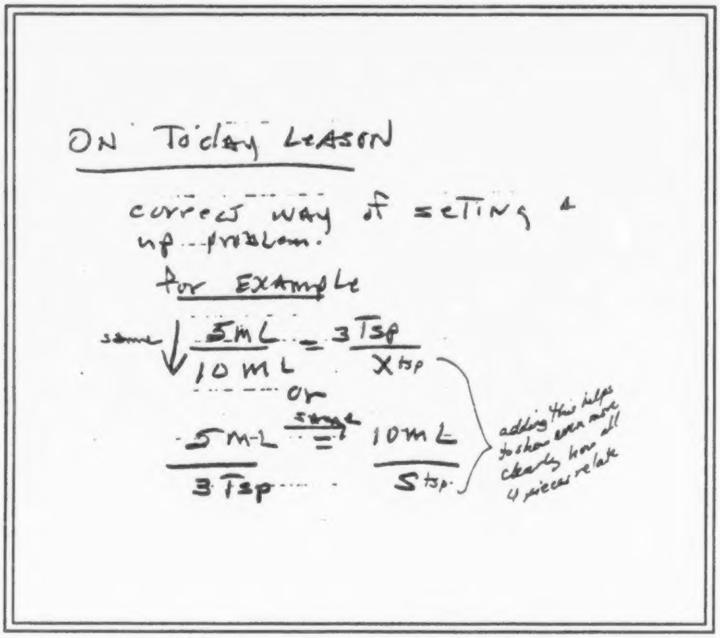
The student who entered the following journal jotting now has a sense for metric measurements of length. I feel that he will remember these estimates since he has written about them.



I LIKE HOW THE CONVERSIONS SET UP IN A RATIO BECAUSE IT'S GETTING BACK TO ALGEBRA FOR SOLVING FOR X. (IE. X COULD LITERS, GRAMS, ECT.) I CAN RELATE TO THAT MUCH EASIER. ALSO IT'S A NICE WAY TO CHECK IF YOU ARE CORRECT IN DOING YOUR PROBLEM. ALSO RELATING 1CM TO 1/2 AN INCH, 1 METER IS ABOUT 1 YARD HELPS TO KNOW IF YOU ARE IN THE BALL PARK WITH YOUR ANSWERS.

I love "solving for "x" also.

Not only did the journal writing allow me a glimpse of how the students used the language of math, it also became a tool for assessment. When students provided examples such as those below, I was able to determine what the students were taking in during the class sessions. In the journal jotting below, the student added arrows to explain how to set up a proportion. I had not used any arrows in my discussion.



ON To'day LeASON

correct way of seting a up problem.

for EXAmple

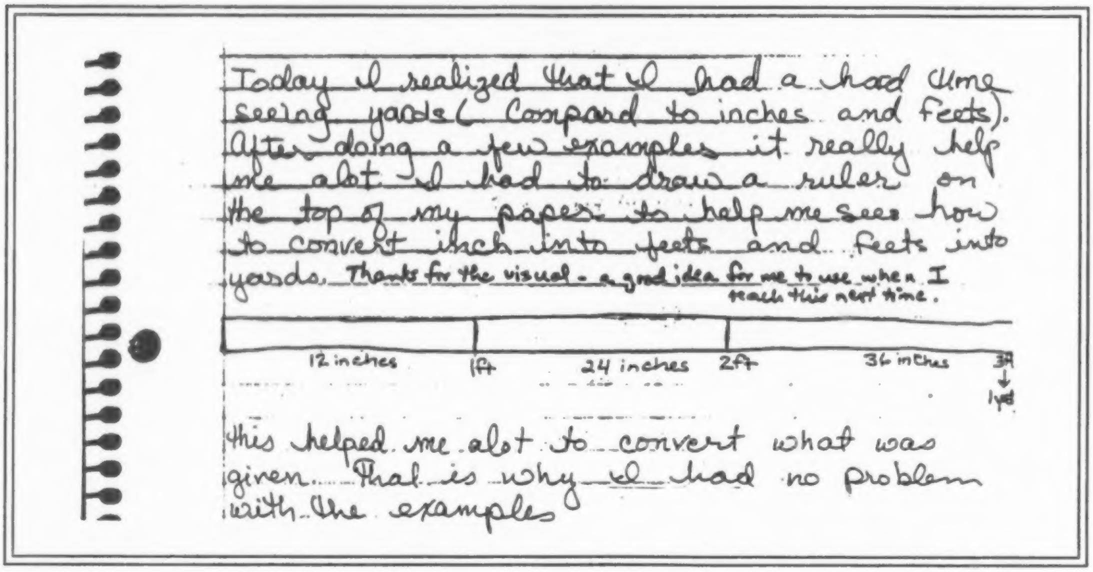
same ↓ $\frac{5\text{ mL}}{10\text{ mL}} = \frac{3\text{ Tsp}}{X\text{ tsp}}$

or

$\frac{5\text{ m-L}}{3\text{ Tsp}} \overset{\text{same}}{=} \frac{10\text{ mL}}{5\text{ tsp.}}$

adding this helps to show even more clearly how all 4 pieces relate

The student below gave me an indication that she was now more comfortable with the relationship among inches, feet, and yards. She used the journal as an opportunity to check herself on her new knowledge.

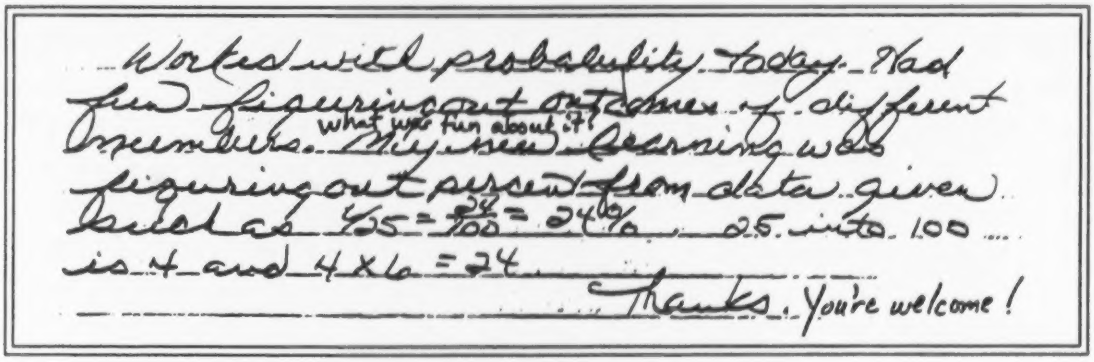


Today I realized that I had a hard time seeing yards (Compard to inches and feets). After doing a few examples it really help me alot I had to draw a ruler on the top of my paper to help me sees how to convert inch into feets and feets into yards. Thanks for the visual - a good idea for me to use when I teach this next time.

12 inches 1ft 24 inches 2ft 36 inches 3ft ↓ 1yd

this helped me alot to convert what was given. That is why I had no problem with the examples

The individual who jotted the note below did not enjoy writing. This entry showed me that she was beginning to expand and give specific examples. In earlier journal notes, she would have ended with "*... figuring out percent from data given.*"

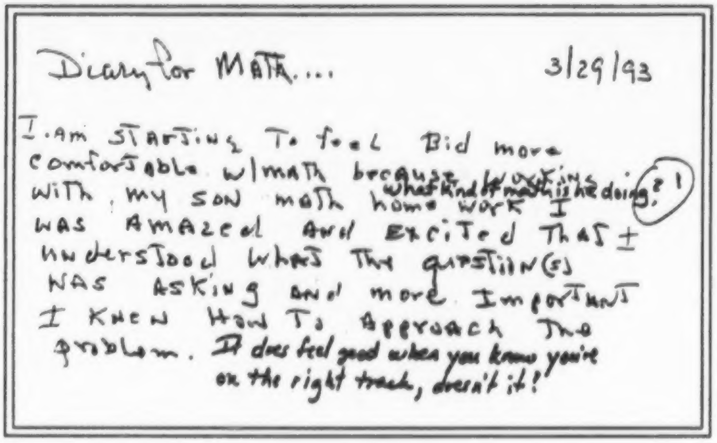


Worked with probability today. Had
fun figuring out outcomes of different
numbers. what was fun about it? My new learning was
figuring out percent from data given
such as $6/25 = \frac{24}{100} = 24\%$ 25 into 100
is 4 and $4 \times 6 = 24$.
Thanks. You're welcome!

Connection to Family

An unanticipated outcome from the content of the journals was the connection to the family. Several times throughout the cycle, students mentioned that they were trying various activities at home with their children. Not only were they attempting activities (especially those in which manipulatives were used), but students also mentioned that they felt more confident in helping their children with their math homework.

I find this particularly interesting since we as adult educators tend to create artificial distinctions between different types of adult literacy: workplace vs. family vs. GED vs. ABE. The following examples show how students in the workplace have made a connection between "workplace education" and "family literacy."



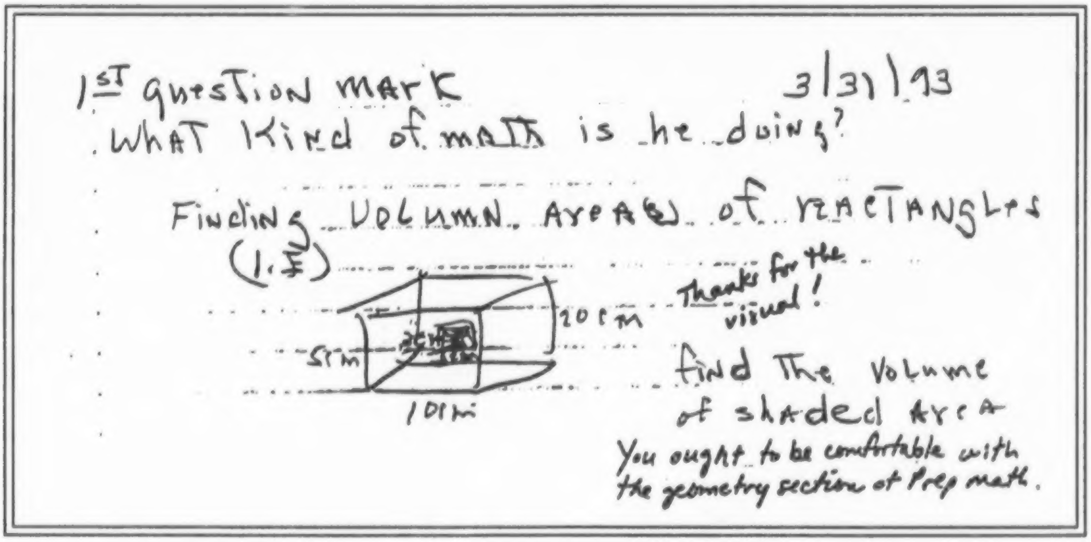
Diary for MATH.... 3/29/93

I am starting to feel Bid more comfortable w/math because working with my son math home work I was amazed and excited that I understood what the question(s) was asking and more important I knew how to approach the problem.

What kind of math is he doing? ?!

It does feel good when you know you're on the right track, doesn't it!

In the above journal jotting, I encouraged the student to discuss math in more detail. When probed about the kind of math he was teaching his son, he replied:



1st question mark 3/31/93

What kind of math is he doing?

Finding volumn area(s) of reactangles

(1.F)

20cm
5cm
10cm

Thanks for the visual!

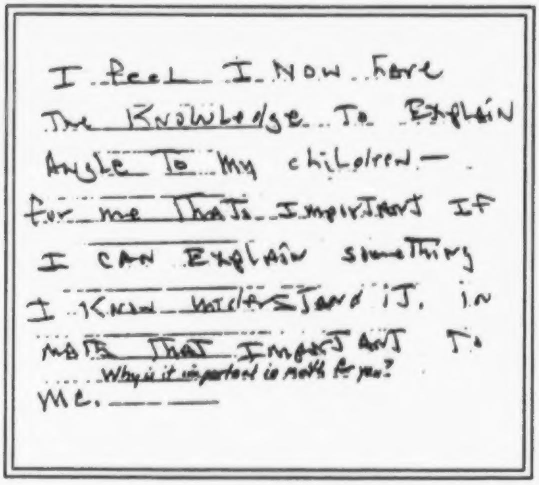
find the volume of shaded area

You ought to be comfortable with the geometry section of Prep math.

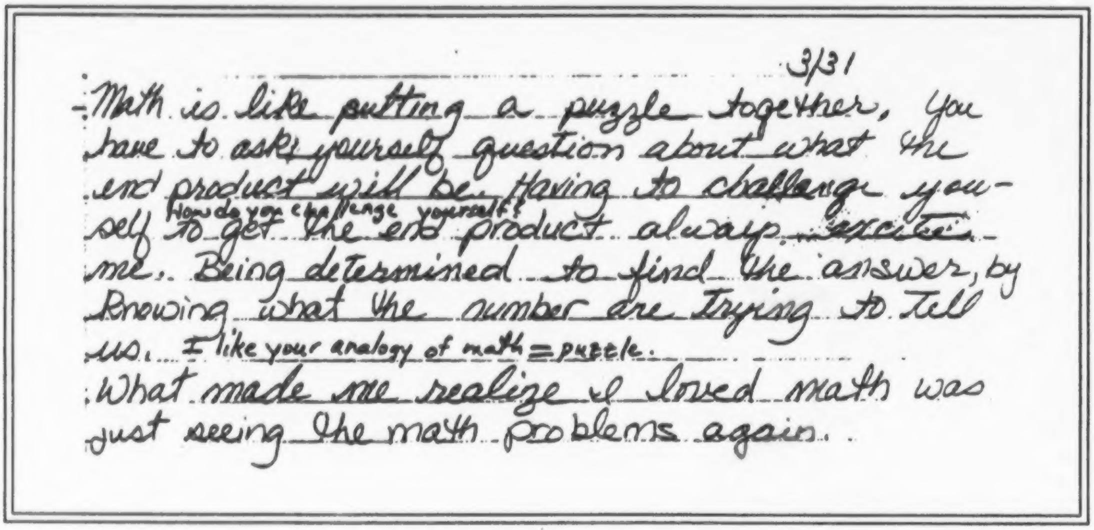
The student below articulated one of the real values of communicating math: when we talk about what we learn, when we share with others, we gain a deeper understanding.

Math as a Challenge

On many occasions students wrote that math was challenging. This recurring theme was true for all students participating in the class. As an instructor this sense of challenge was reassuring. Students seemed to feel that math was not something negative; more it was seen as something that they could rise up to and meet without fear. I suspect, though, that the term *challenge* had a different connotation for each student, and individuals looked at each topic as a different form of *challenge*.

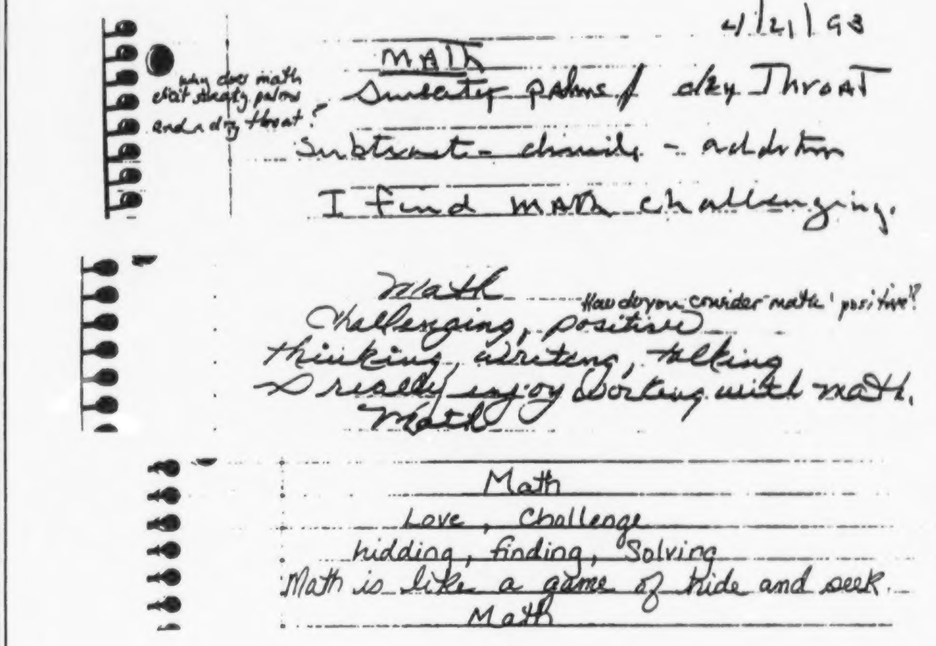


I feel I now have
The Knowledge To Explain
Angle To my children—
for me That's Important If
I CAN Explain something
I Know understand it. in
MATh That Important To
Why is it important in math for you?
me. ——



3/31
Math is like putting a puzzle together, you
have to ask yourself question about what the
end product will be. Having to challenge you-
How do you challenge yourself?
self to get the end product always excite
me. Being determined to find the answer, by
knowing what the number are Trying to Tell
us. I like your analogy of math = puzzle.
What made me realize I loved math was
just seeing the math problems again.

On one occasion during the project, students were asked to write a five-line poem about math. (The first and last lines consisted simply of the word "math". The second line consisted of two adjectives describing math, the third line three verbs, and the fourth line was a sentence about math.) Below are three examples, all revealing students' thinking about math as a challenge.



4/21/93

MATH

Sweaty palms & dry Throat

Subtract - divide - addition

I find MATH challenging.

why does math elicit sweaty palms and a dry throat?

Math

Challenging, positive

thinking, writing, talking

I really enjoy working with math.

Math

How do you consider math 'positive'?

Math

Love, Challenge

hidding, finding, Solving

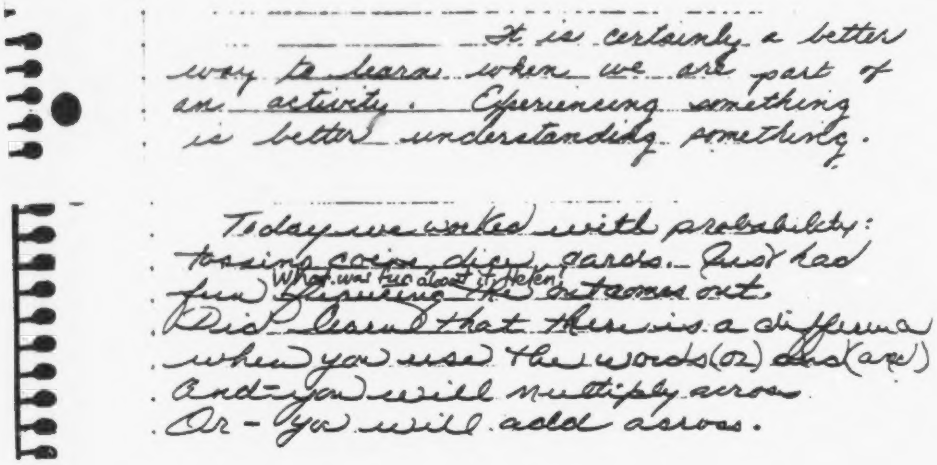
Math is like a game of hide and seek.

Math

Use of Manipulatives

The discussion of using manipulatives with adults has been a topic of conversation with many of my fellow Math Team members. Comments made by students in their journal jottings confirmed for me that they do enjoy using manipulatives, and I definitely intend to continue to incorporate them into any math class I teach—at whatever level of instruction.

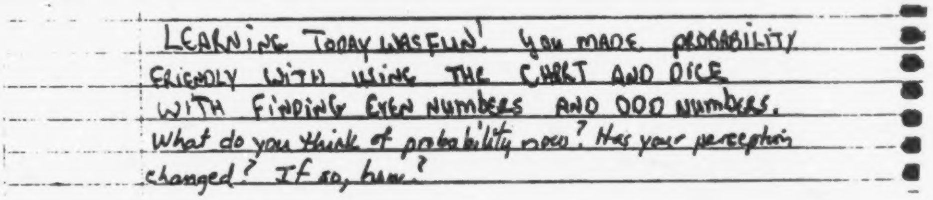
What kind of manipulatives did I use? I used quite a few: We folded and tore cut-out triangles to prove there are 180 degrees in a triangle. We measured different size circles to determine that the circumference was always a little bit larger than three times the diameter (pi). We used dice when first learning about probability and later in the discussions around theoretical vs. experimental probability. This was a nice segue into sampling used at the work site. We counted the number of candies in mini-bags of M & M's to create histograms, and we used the same bags to talk about sampling and the probability of choosing various colors.



It is certainly a better
way to learn when we are part of
an activity. Experiencing something
is better understanding something.

Today we worked with probability:
tossing coins, dice, cards. Just had
What was fun about it, Helen?
fun figuring the outcomes out.
I did learn that there is a difference
when you used the words (or) and (and)
And - you will multiply across
Or - you will add across.

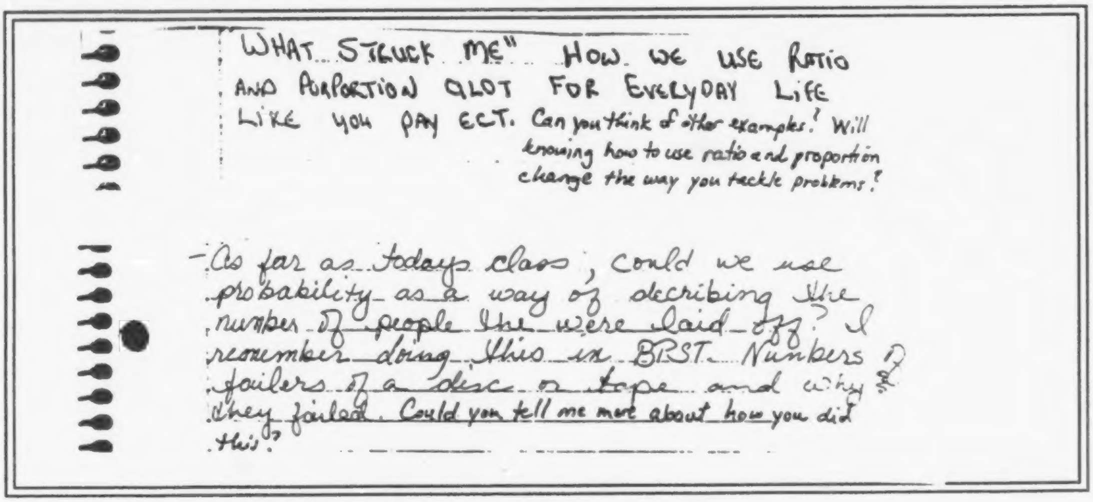
On several occasions, students commented on having "fun". I think the manipulatives helped to bring fun into the classroom. I also believe that learning is facilitated when fun is involved...but that's a question for yet another research project.



LEARNING TODAY WAS FUN! YOU MADE PROBABILITY
FRIENDLY WITH USING THE CHART AND DICE
WITH FINDING EVEN NUMBERS AND ODD NUMBERS.
What do you think of probability now? Has your perception
changed? If so, how?

Connection to Daily Life

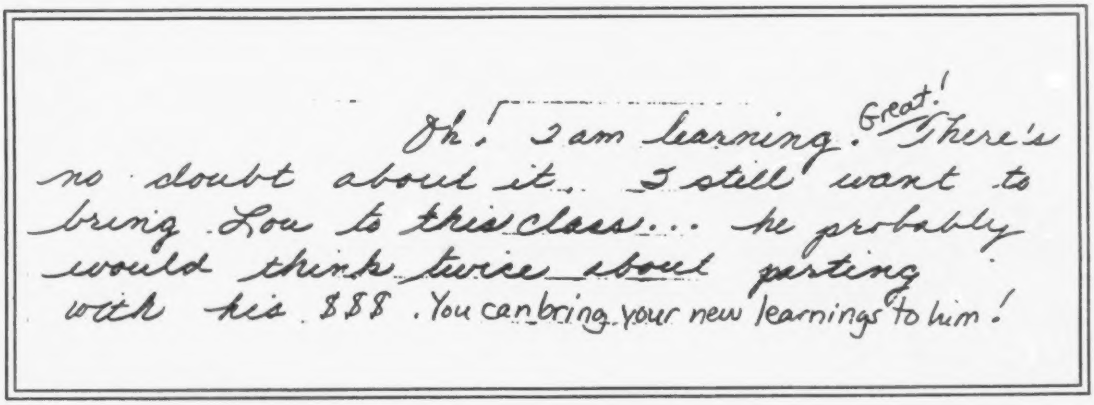
Whenever possible, I tried to connect math to daily life as well as to the work environment. It was refreshing to see that students were also making the connections.



WHAT STRUCK ME" HOW WE USE RATIO AND PORPORTION ALOT FOR EVERYDAY LIFE LIKE YOU PAY ECT. Can you think of other examples? Will knowing how to use ratio and proportion change the way you tackle problems?

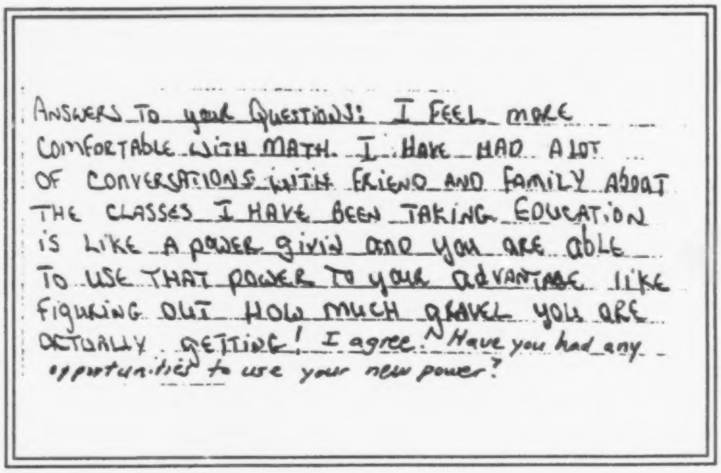
-As far as todays class, could we use probability as a way of decribing the number of people the were laid off? I remember doing this in BRST. Numbers of failers of a disc or tape and why they failed. Could you tell me more about how you did this?

Not only did students think about applying concepts in areas outside the classroom, they also communicated their learning to others.



Oh! I am learning. Great! There's no doubt about it. I still want to bring Lou to this class... he probably would think twice about parting with his $$$. You can bring your new learnings to him!

I believe journal jottings can be valuable tools for instructors. Comments such as the one below gave me insight into how the student makes connections to the real world as well as the value he places on education.



ANSWERS TO your QUESTIONS: I FEEL more COMFORTABLE WITH MATH. I HAVE HAD A LOT OF CONVERSATIONS WITH FRIEND AND FAMILY ABOUT THE CLASSES I HAVE BEEN TAKING EDUCATION IS LIKE A power givin and you are able TO USE THAT power TO your advantage like FIGURING OUT HOW MUCH gravel YOU ARE actually GETTING! I agree. Have you had any opportunities to use your new power?

CONTENT OF INTERVIEWS

One of the most enjoyable pieces of this research project was the opportunity to interview each student. I decided to include interviews as part of my project because I wanted to find out what students thought about the process of writing.

The students all agreed that journal writing was a new experience for them, not one that they at first readily accepted. One student stated,

"At first it felt awkward because I had never done it before."

When asked if he would recommend journal writing again, he replied,

"ANY class should have journal writing. I feel people will not say anything out loud in a group. I see people all the time be afraid to say how they feel. This way they can."

Another student responded that the process of writing in a journal was *"... quite different"*. She stated, *"I'm a phone person."* She further added that journal writing was a good experience because *"It is good to write down what is new. You forget new words learned during the day."* She felt, though, that it would have been more effective to write down things all during the class time because *"I forget what I want to say. I forget at the end."*

This student's comment reminded me of why I began this research project, that is, the mistaken belief held by whole language experts. When asked how she felt about journal writing in a math class, she stated,

"I usually can express myself better on paper, but I found it strange to have to talk about math. I had difficulty writing about math."

When asked to further elaborate, she said that she likes to write because she can be creative, but in writing about math,

"Well, math is math! Math is finding the answer and that's it. Show me the techniques and I'll solve the problem. What's to write about?"

Fortunately, she went on to say that she has changed her thinking about math and writing about it. She later added,

"By writing about it, it helps me to remember."

A final comment I found most interesting was this:

"It's almost like telling someone a secret without fear of wondering what someone else thinks. You gain a trust through writing about it."

CONTENT OF ASSESSMENT

The results of the assessment did not related directly to the process of journal writing, but I found some possible connections between comments made in students' writing and outcomes of the assessment.

Students did comment in their journals that they were making connections to everyday life. Several statements on the assessment also referenced home activities, so I was able to make some pre/post-assessment comparisons.

Statement #8: *"Probability is useful in my daily life, both at home and at work"*, for example, showed a significant increase.

Statement #6: *"I use geometry when I'm working on projects at work"* also showed an increase.

In having the students participate in journal jottings, I wanted them to become more comfortable with the language of math. On the post-assessment, Statement 5: "I am comfortable when people around me are talking about the metric system" reflected a significant improvement from the pre-assessment. Another statement about the metric system, #11: "I am comfortable solving problems involving the metric system" also improved.

I received some interesting post-assessment comments on open-ended statement H: *"I think writing in a journal is ____"*:

"...making you think when I don't want to!"

"...a communication tool for teacher and student."

"...a good way to express your feelings."

The entire assessment form, with pre- and post-comparisons, is in the appendix.

UNFINISHED BUSINESS

How will all this information impact my teaching? I plan to continue journal jottings in math classes, at all levels of math. I feel that the journal was a way for students to express themselves, especially those who may be reticent to speak up in class. I also believe that students, in writing about what they were learning, became more fluent in the language of math.

Although the results of this research project confirmed for me that writing does have a place in the math classroom, I am left with several questions regarding journal writing:

✓ **How can I encourage students to write more?** Even though I phrased my response questions to encourage students to explain their ideas more fully, I still had some who would write as little as possible.

✓ **How do I make sure students are not just writing what they think I want to hear?** I had established excellent rapport with my students; many I knew already from earlier classes and from visits at the worksite. I wonder if, because they did not want to "offend" me, they sometimes did not share any "negative" feedback about class.

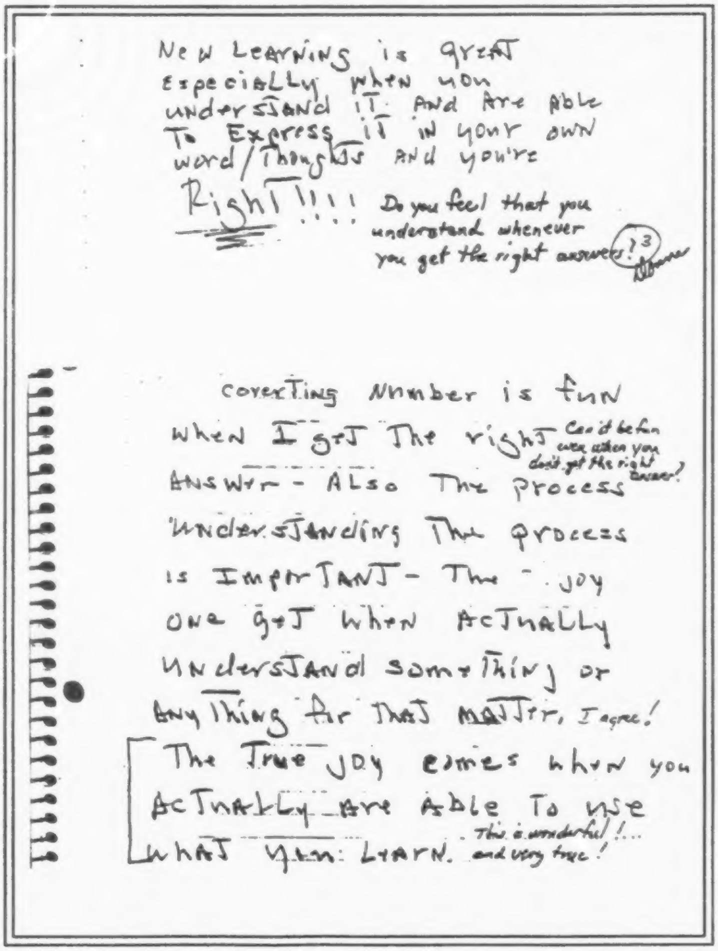
✓ **How can I tell what specific activity helped students gain confidence and learn more effectively: manipulatives, style of presentation, journal writing?** Obviously, this question begs for more research projects!

Another question relates to the outcomes from the assessment:

✓ **Did students really begin to use math more at home** (based on the responses to statements 6-8), **or did they just *realize* that they were already using math?**

Reading the journal of one individual student left me with a larger question:

✓ **How do I help the student who is afraid of being wrong?**



New Learning is great
especially when you
understand it and are able
To Express it in your own
word/Thoughts and you're
Right!!!!

Do you feel that you
understand whenever
you get the right answers? ?3

coverting Number is fun
when I get The right
Answer - Also The process
understanding The process
is ImportanT - The joy
one get when Actually
understand something or
anything for that matter,

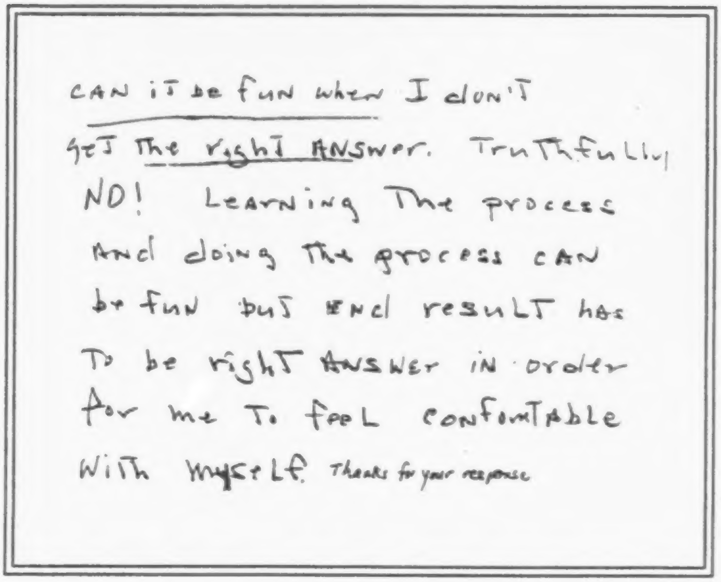
Can it be fun
even when you
don't get the right
answer?

I agree!

The True joy comes when you
Actually are Able To use
what you Learn.

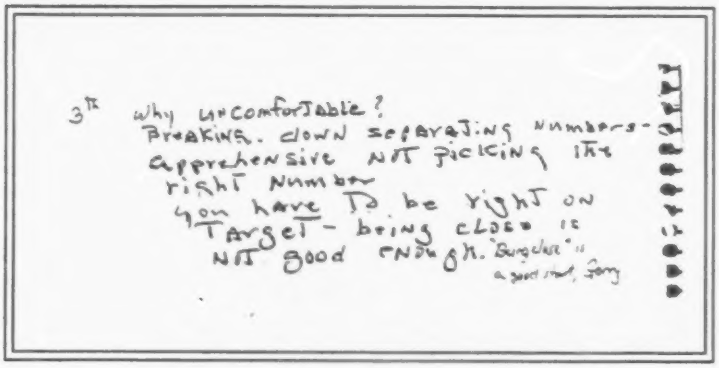
This is wonderful!...
and very true!

Here is the response to the question I asked on the previous page:



Can it be fun when I don't get the right answer. Truthfully NO! Learning the process and doing the process can be fun but end result has to be right answer in order for me to feel confortable with myself. Thanks for your response

This last comment, from the same student, was in response to my question regarding the content of his poem on math. As an instructor trying to instill joy in learning math, I found his comments disturbing.



3rd why uncomfortable?
Breaking. down separating numbers - apprehensive not picking the right number
you have to be right on target - being close is not good enough. "Burgclose" is a good start, Gary

In the workplace and in real life, adults combine a variety of skills to accomplish tasks. So, too, should the case be in an adult education class. Rather than isolating math from writing or reading or any other content area, content areas should be interrelated to effectively facilitate learning.

To answer to my original question:

"Do students gain a better understanding of and broader insight into math concepts if they write about what they are learning?"

would have to be YES. Based on comments made by students and my own experience on this project, I firmly believe journal jotting does have a positive effect on students learning math.

Following is a copy of the assessment given both at the beginning and at the end of the course. I averaged the responses for a pre- and post-assessment comparison. The pre-assessment averages are circled, while the post-assessment averages are boxed in. To get a truer sense for the results, I only included the responses of the students who participated in both assessments.

APPENDIX

PREP MATH
MATH ASSESSMENT

Name ______________________________

Date ______________

Rate each of the following statements:

1. I become anxious whenever I have to work a problem involving math.

(A) 1 2 3 4 5 6 7 8 9 10
Never Sometimes Always
6.7
6.7

2. My stomach churns whenever I am asked to participate in an SGIA*.

(B) 1 2 [✓ 3] 4 4.7 5 6 7 8 9 10
Never Sometimes Always

3. I have thought about taking a college level math class.

(B) 1 2 3 4 5 [✓ 6] 7 8 9 10
Never Sometimes Always
6

4. When I read, I tend to skip over any charts or graphs in the text of the material.

(B) 1 2 3 4 5 6 7 8 9 10
Never 3.3 Sometimes 6.3 Always

5. I am comfortable when people around me are talking about the metric system.

(A/K) 1 2 3 4 5 [✓ 6] 7 8 9 10
Never 3.3 Sometimes Always

6. I use geometry when I'm working on projects at work.

(B) 1 2 3 4 5 6 7 8 9 10
Never 2.3 Sometimes Always
3.3

7. I use geometry when I'm working on projects at home.

(B) 1 [✓ 2] 3 4 5 6 7 8 9 10
Never 3.7 Sometimes Always

**SGIA stands for Small Group Improvement Activity, an acronym used by the company to signify a team project.*

8. Probability is useful in my daily life, both at home and at work.

(K) 1 2 3 [✓ 4] 5 6 7 8 9 10
Never Sometimes Always
6.7

9. When I go shopping, I try to figure out in my head how much I am saving on sale items.

(B) 1 2 3 4 5 6 7 8 9 10
Never Sometimes Always
8.3
8.7

10. I am willing to keep track of the household budget for my family.

(B) 1 2 3 4 5 6 7 8 [✓ 9] [✓ 10]
Never Sometimes Always

11. I am comfortable solving problems involving the metric system.

(K) 1 2 3 4 5 6 7 8 9 10
Never Sometimes Always
3.7
6.3

12. Fractions "intimidate" me.

(A/K) 1 2 3 4 5 6 7 8 9 10
Never Sometimes Always
3.3
3.7

A. When I hear the word "math", I ______________________________.

B. I think probability is most useful in ______________________________.

C. When I see a problem involving a fraction, I ______________________________.

D. If you asked me what 25% of a number is, I would ______________________________.

E. To me, "volume" means ______________________________.

F. I use a calculator whenever I have to ______________________________.

G. When I hear someone talk about the "average" of something, I think ____________________.

H. I think writing in a journal is ______________________________.

I. What I hope to get out of this Prep Math class is ______________________________.

SHAPES AND STITCHES: QUILTING IN AN ABE MATH CLASS

By Linda Huntington, Community Learning Center, Cambridge

I think that finding a question was the hardest part of my research project. I felt it was important to balance finding a project I was excited to undertake *and* the need to focus on a content area that was either vital to students better understanding mathematics, or that was traditionally ignored in teaching ABE students (or both). I hoped as well that the project would address the *NCTM Curriculum and Evaluation Standards for School Mathematics*. In particular, I hoped that it would affirm my commitment to stressing communication as a key to learning math content and skills. Algebra and geometry are often overlooked by teachers in ABE math classrooms. I think many teachers cling to the belief that math learning progresses in a linear order, with geometry and algebra coming at the end of the progression. As a result, many ABE students are never given the chance to even play with algebraic notation, missing numbers in problems, or angle measurements. Many, especially lower level ABE learners, cannot even recognize the names of common geometric shapes.

"In the adult basic education mathematics curriculum, importance should be placed on providing learning activities which reflect the concepts and properties of geometry itself: the use of 3-dimensional hands-on models and problem solving experiences that allow for physical exploration of the effects of change in angle and measurement."

MASSACHUSETTS ADULT BASIC EDUCATION MATH STANDARDS, STANDARD 10: GEOMETRY AND SPATIAL SENSE

An idea came to me one day over lunch conversation. I would love to make a quilt in one of my math classes. A question began to form: What kinds of mathematical skills do students use when they make a quilt in math class? Yes, I knew that I would enjoy doing this project. Making a quilt would definitely include geometry, measurement, money, fractions, and a lot of problem-solving. The connection between art and math should become obvious, as would connections between math and real life skills like sewing and measuring. And I felt certain that a group of students quilting together would communicate with and help one another without hesitation.

I knew that the idea of making a quilt in math class would be new and different, but I did worry about how a group of ABE students would accept the project as part of learning math. Because of this uncertainty, I had to carefully choose which of my classes to work with. I eliminated my GED group; they were fast approaching the time of testing and definitely would not want to make a quilt. In the end I decided to choose the beginning level ABE class (reading level 0 - 4.7) for several reasons: they had a strong tendency to cooperate and help one another; they had recently been exploring number relations and shapes with pattern blocks; they were not in any way test-driven; they had three math classes a week, for a total of 4 1/2 hours of math; geometry as a content area was seldom attempted in the lower level ABE class. I also had a gut feeling that as a group they had strong visual skills, which, if reinforced, might strengthen their understanding of math. When I asked if they wanted to make a quilt in math class, they agreed.

The only problem I anticipated when choosing this group was one of numbers. There were only six students in the class when we began the project. And shortly after we began, two students tested out of the class into a higher level. Like all ABE classes, attendance would be an obstacle. Undaunted by the drop in numbers and the possibility of attendance problems, we forged ahead. We decided that instead of making an entire quilt, we would just make individual quilt squares that could be wall hangings or maybe pillow covers. Our first of many experiences with the dynamics of group problem-solving! Thus we became a core group of five (including teacher) quilters *aka* math problem solvers.

The group, although small, is diverse. One student is from Venezuela, one from Haiti, one from Ethiopia, and the other from the USA. Of the four students, only one had much experience sewing, one had considerable artistic talent, one had a keen ability to perceive patterns in architecture and sculpture, and the fourth, a definite and bold sense of color as seen in her clothes and later in her choice of fabric. Also a consideration I now know to be of more importance than I thought at the time, is that the four students have very different work styles. These work styles pervaded their approaches to both problem-solving and to their success at quilting. I already knew from teaching math to the group that Sara is a very serious, aggressive and conscientious worker; that

A consideration I now know to be of more importance than I thought at the time, is that the four students have very different work styles.

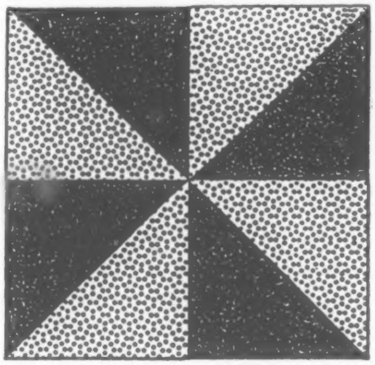
Eddie is very impulsive but not systematic in his follow-through; that Marie is much more laid-back and passive when approaching her learning; and that Jorge often seems to lack direction, but has sudden profound insights. He is full of surprises!

My beginning thoughts were to work on the quilt project two of the three 1-1/2-hour math classes each week. This would give the students one class every week to continue practicing the basics of computation in a familiar class structure. I hoped to incorporate into their computation practice some of the math they learned in their quilting classes. Further into our project, and closer to the end of the semester and writing deadline, we often used all three classes to work on the quilt. Seeing the finished quilt square became a strong incentive for spending increased time sewing. Some days I could hardly get them to leave math class to go to their reading class!

All four students attacked the task of choosing and making quilt squares in their own unique manner. Predictably, three of the four students' approaches mirrored their classroom workstyles. In the case of the fourth student, her manner was both typical and atypical when compared to her usual math persona. I found this a little puzzling. I want to briefly describe each student's method of attack followed by a picture of his/her quilt square and some words of their own to describe their square.

Marie looked quickly through the book showing quilt patterns and immediately settled on a pattern which only required one triangle shape repeated in two different colors. I think that for Marie the choice of an easier pattern was crucial. When we had spent class time talking about and copying shapes, Marie had a lot of difficulty reproducing specific shapes—she could recognize them, but she could not easily copy them. The contrast of light and dark colors is what made Marie's pattern work, not a play of several shapes. She decided from the time of cutting paper shapes to make her quilt red and yellow, colors she often likes to wear. Marie went to the fabric store and directly chose two fabrics— one red, one yellow. When it was time to sew, she started right in. I asked her if she had ever sewn before and she replied no. But she was able to learn easily how to both baste and sew on the machine with minimal effort. Marie would watch my sewing demonstration without asking a question, and then she would quietly do it herself. She just sewed, sewed, and sewed until she finished—weeks before anyone else was done! She did not spend time worrying over the finer points of sewing technique. If a seam was a little crooked, or if the corners did not exactly meet, Marie did not care, she sewed on, enjoying every minute of her work. When she finished, she asked to help everyone else. As Marie worked on a second pattern, she began to perfect her sewing technique, and really had fun learning how to become a better seamstress.

Marie's square:

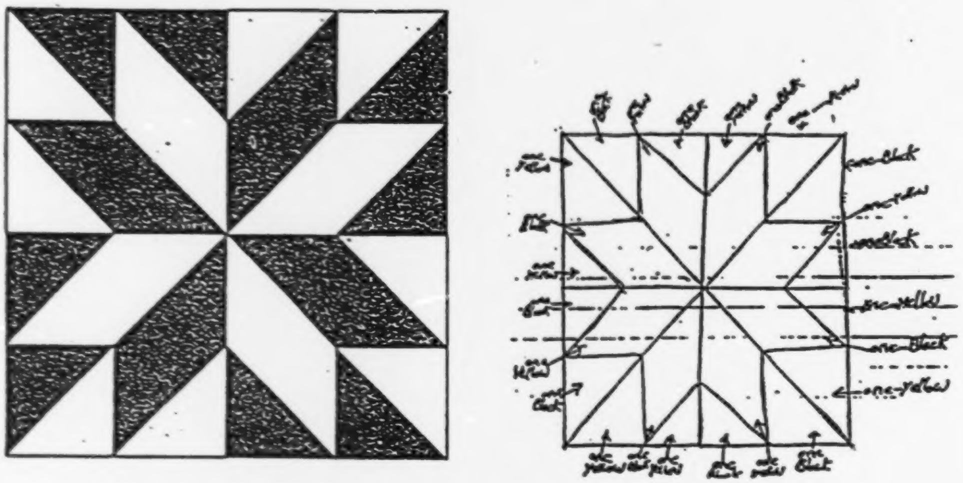


My quilt square is named windmill.

It is made of eight triangles.
Four are yellow and four are red.
I put one red. After that I put one yellow
I repeated my pattern four Times in a
To me The picture looks like a kite.

BEST COPY AVAILABLE

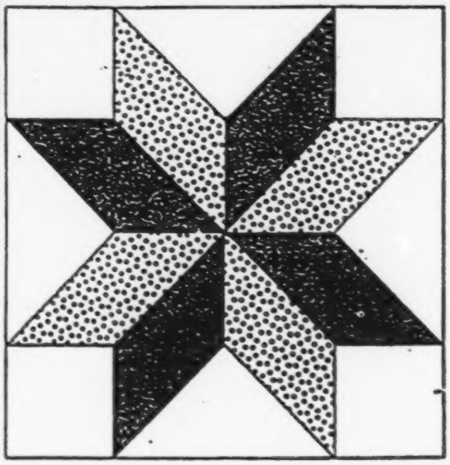
Eddie spent much of his time at school doodling and drawing patterns. I knew he would like the idea of making a quilt. He took some time looking at quilt pictures before he chose one he liked. Eddie's choice was a complicated pattern. It only required two shapes — a triangle and a parallelogram. The complications arose from the manner of piecing together the different colors and shapes. Right from the start, Eddie was confident that making the square would be easy and fast for him. He also was sure that he wanted to use only two colors — gold and black. Eddie's artistic nature enabled him to easily draw his pattern and cut and paste the two shapes and colors onto paper accurately. He was able to see that his pattern was virtually the same as Jorge's, and he was a tremendous help when Jorge was cutting both paper pieces and material. Eddie was able to fold paper and cut multiple pattern pieces at once. He showed the others in the class how to follow his technique. When it came time to buy material, Eddie changed his color plan entirely—he chose four different colors of material instead of two—neither black nor gold was included. Eddie knew how to sew by hand, so he jumped right in basting his pieces. He learned to use the sewing machine, and again just jumped in with two feet. Eddie was not too interested in perfection of sewing. In fact, when it turned out that he had cut two parallelograms of each color incorrectly (upside down), making it impossible to sew them together in the exact pattern, he just decided to change the pattern to fit the pieces rather than cut the pieces over.



My quilt square is May 7, 1993
Named. Stars and Stripes
Ii is made of PARALLELOGRAM. - 8 -
TRIANGLE - 16.
8 yellow - 8 Black.
4 yelbw - 4 yellbw

Eddie's square:

Jorge seemed uncertain of what was intended by making a quilt square. He really liked all the exercises using the pattern blocks. He seemed to understand and enjoy learning about various geometric shapes. But he appeared unable to put it all together. After we all went together to shop for material, Jorge asked me, "Why did we just buy all that material?" Jorge (not surprisingly) had difficulty first drawing his square, and then cutting paper pattern pieces. He got a lot of help from Eddie, Sara and Marie at every step of the way. Jorge had never before sewn either by hand or on a machine. He made several allusions to having skipped this class in his previous schooling. He said it had never been of interest to him to learn to sew. But now it was. He told me,



Jorge's square

My quilt square is NAMED: STAR OF LE MOYNE. IT is MADE OF EIGHT PARALLELOGRAMS. FOUR ARE GREEN and FOUR are Red. THERE ARE FOUR YELLOW TRIANGLES. THERE ARE FOUR BLUE squares. THE PARALLELOGRAMS ARE IN THE MIDDLE. THEY LOOK LIKE A STAR SHAPE. THE FOUR squares are in EACH CORNER. THE YELLOW TRIANGLE are on THE SIDES IN BETWEEN THE PARALLELOGRAMS and THE squares.

"Thank you for teaching me how to sew. Now I can fix my own pants and save $6.00." He was afraid of the sewing machine at first, but improved with practice. When cutting his pattern on paper, Jorge wanted to use four colors—red, blue, yellow and brown. At the fabric store he finally decided on four colors— indeed blue, red (magenta), yellow and brown. His idea of color combination surprised me and the other students, but he persisted. And the colors when finished (or at this time, when almost finished) looked amazingly good together.

The three previous students and their work, their attitudes, their thinking and writing did not surprise me. But **Sara**, from the day the project began, showed me a different side of herself.

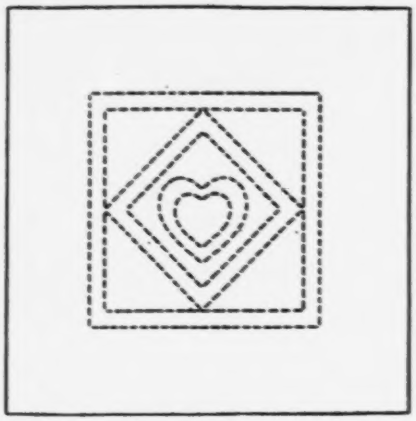
Sara looked through several books of quilting patterns. It took her a long time to decide on a pattern. Her pattern required only two colors, but it was much more complicated than Marie's choice. Each time I asked the group to draw their quilt square, Sara left hers unfinished. I puzzled over this because Sara had always been the most serious and the most mathematically confident and quick in the class. She always completed both classwork and homework. When asked to describe her square, she had much more difficulty coming up with the needed words than I thought she should.

Sara is usually an outspoken, and a bright student. She never before seemed to be at a loss for words. Sara sewed meticulously, but slowly. Her care did not surprise me, but her speed did. When something did not come out perfectly, she took out the stitching and did it over. Although she smiled and talked with other students while she sewed, I had the distinct feeling that Sara was not enjoying the quilting experience. I was beginning to worry that she was feeling that quilting just was not mathematics.

I had the distinct feeling that Sara was not enjoying the quilting experience. I was beginning to worry that she was feeling that quilting just was not mathematics.

One day Sara was the only student in class, I asked her if she liked the quilting project. She said she did not like to sew. When I asked her why, she replied, 'My mother sews for a job in my country. She is a sewing teacher. When I was little she taught me how to sew and I never liked it." One day when she was a little careless with a seam, Sara said, "My mother would not have let me sew like that, she would make me take it out and start over again." I began to understand why Sara was spending so much time sewing perfect seams. On the other hand, I stopped worrying that she would finish her square because there was a kind of determination in the way she was working. This was more the Sara I knew from math classes.

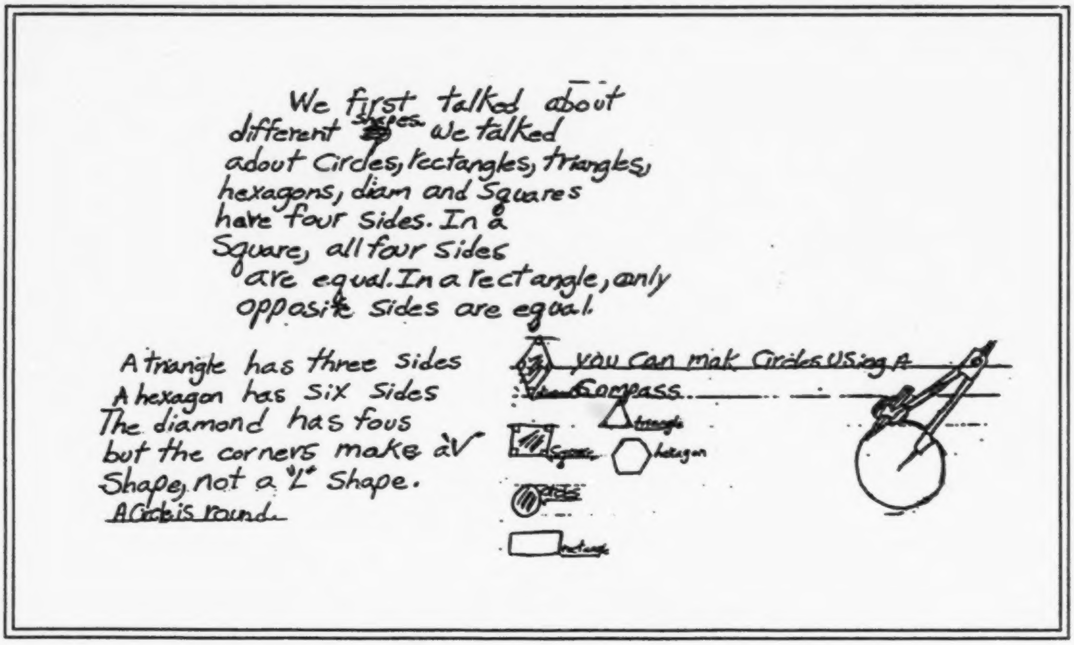
My quilt square is called
Diamonds in the square..
Around the outside are four rectangles.
In the middle of the rectangles.
is a square, and in the middle of the square
is a diamond.



TIME SPENT

I roughly divided the content of classes during the project into three parts. During the first part I introduced the idea of quilting to the group. We talked about shapes in nature and man-made shapes in the world around us. We played with pattern blocks, comparing different shapes, and making pictures of shapes. During these first classes, the students learned vocabulary, looked at examples of quilts in books, and chose the pattern they wanted to make. The middle classes were sort of the preparing-to-sew classes. Students drew their squares, made paper quilt squares, talked about color, made templates and shopped for their fabric. They also practiced hand stitching and learned to sew using the sewing machine. The last third of our classes was devoted to the actual sewing of the quilt squares. We took one class out of our sewing time to look at a videotape telling stories from the AIDS quilt.

At different times throughout the project, students wrote what they had learned. Our writing process was usually done collectively, following a kind of language experience method of writing. The students would together dictate, I would write on the board, and then they would copy "their" words. This is what they wrote after learning about various shapes. This is a copy embellished by Eddie. It is easy to see how this exemplifies his drawing abilities.


triangle
Square
hexagon
circle
rectangle

We first talked about
different shapes. We talked
adout Circles, rectangles, triangles,
hexagons, diam and Squares
have four sides. In a
Square, all four sides
are equal. In a rectangle, anly
opposite sides are equal.

A triangle has three sides
A hexagon has six sides
The diamond has fous
but the corners make a "V"
Shape, not a "L" Shape.
A Circle is round.

you can mak Circles Using A
Compass.

Toward the end of the semester I took aside each student one at a time and asked them to answer some questions for me. I had written a short questionnaire that I felt they needed to answer so I would know what they had learned and their feelings about our project. After answering the questions I also asked them to match shapes to their respective names. Here are the questions I asked my class:

1. What did you like best about sewing a quilt square?
2. What did you like least about sewing a quilt square?
3. Do you think you would like to sew another quilt square?
4. Would you like to make a quilt that had picture squares?
5. What new skills did you learn? What can you do now that you could not do before?
6. Did you learn: how to measure with a ruler?
    the names of some gemoetric shapes?
    how to divide 8 inches in half?
    the difference between horizontal, vertical and diagonal?
7. What other math did you learn?

WHAT WE LEARNED

Did our project (this really was much more a group project than an individual one) address the desired *NCTM Standards*? The Standards I expected to include were the first four: *Math as problem-solving, math as communication, math as reasoning, and mathematical connections.* Reasoning skills are used when problems are solved. As problems arose in making quilt squares, the students reasoned and questioned, discussed with the group, and tried their answers until the problems were solved. As the weeks of the project went by I observed a lot of mathematical insights, and saw a lot of real thinking and problem-solving.

The students found themselves in a number of situations which required them to determine the answer to one problem before work could continue. In some cases the problems were easy to solve. When Eddie wanted to know which pieces to join together first, and which sides of his parallelograms went together to make the pattern work, he only needed the time to look at the quilt square made of paper and then move the fabric pieces until they were the same. Then he had to determine either in his mind, or by trial and error (basting and then basting over) what sewing order made sense.

In other cases the problems were not so easy to solve.The quilt square that Sara choose to sew, although visually uncomplicated, squares within squares, became a problem-solving nightmare. To begin with, Sara had a hard time drawing the picture of her square. Hers was the only square where the pattern did not obviously grow out from the center. Even with careful measuring, it was difficult to get all the lines parallel. It took about five tries for Sara to finally draw her square in such a way to make pattern pieces from the picture. When her pattern was made, and the material cut, we made a decision which proved to be disastrous. We decided to sew her pieces together from the outside of the square to the inside.

Poor Sara! After basting and machine sewing her outside four strips, the inside row of strips would not fit. We made another quick decision: we cut the pieces so they would fit. Then when Sara came to the triangular pieces in the middle of the inside strips, they were too big. Once again we tried cutting the pieces down to size; but when we cut the triangles, the diamond in the middle did not fit. By this time, Marie had finished her square. The three of us put our heads together. We realized that all our problems stemmed from the incorrect assumption that even though Sara had drawn her quilt square from the outside-in, it was essential to sew it together from the inside-out.

I was afraid that Sara, the least enthusiastic sewer, would want to give up at this point. She said, "I need to start all over. I have to cut the pieces again from the pattern, and sew them starting with the diamond. Let me take the material home for the weekend." Sara took home her pattern, the necessary material and basted together the entire square over the weekend. When she came into class the next week she said, beaming, "I like it now, it is beautiful."

Connecting mathematics to a craft like quilting gave the students a chance to see that math isn't something to do only once or twice a week in school. For my students math occurred outside the boundaries they had previously applied to the subject. They did in fact make not only the obvious connections, but a few connections I would never have expected.

For my students math occurred outside the boundaries they had previously applied to the subject.

Learners can see that math can be useful in their everyday lives. Sewing is a valuable life skill to know. The students were aware of this, and liked the fact that we could practice this kind of skill in math class. Marie really loved the sewing experience she gained, and maybe she could use her new knowledge one day on a job. Jorge made several interesting connections. He said, "the best part for me is when I used the pattern blocks. I learned more fast math...I learned how to sew. In 30 years I never touched—not even to sew my own pants...The squares were really interesting I saw once in a church."

Sara said, "I never heard the word diagonal, vertical and horizontal before. Now I really learned them because

I do them in aerobics class." This was a surprise to me, so I asked, "How do you use those words in aerobics class?" "In the steps. The teacher would use those words and I never knew what they were talking about. I had to watch the other people and do what they did. Then I learned." Talk about *mathematical connections*!

Communication was the core of our project—listening, seeing, discussing, and sometimes writing. I emphatically believe that communication is the key to successful learning in mathematics. As teachers we need to remember that communication in our classes is not a one way street. It is often difficult to plant the real need for discussion in the students' minds. Sometimes they are afraid to speak out for fear of being "wrong" or just because they don't know where to begin. They need to feel comfortable that in the group it's OK to listen as well as talk, but that everyone should have a time for his or her opinion to be heard. It is extremely helpful for the group to share a common goal. The goal can be as simple as passing a test like the GED. Having a common goal was what made the quilting project so special.

Having a common goal made the quilting project so special.

One part of communication is learning vocabulary. All the students learned some new vocabulary: they learned the words symmetry, diagonal, horizontal,and vertical. They learned the names and properties of specific geometric shapes. They practiced together words of spatial relations: above, next to, to the right/left of, below. One student would make shape pictures and others would copy them from verbal clues without seeing the picture. They realized the importance of carefully choosing descriptive words in order for the audience to "get the picture." They also realized the importance of careful listening skills, often a part of overall communication skills that is ignored by ABE students. I knew the students had learned these concepts and the vocabulary by my own class observations, by their finished quilt squares, by their answers on the final questionnaire, and by their ability to answer questions on a related worksheet.

The importance of having a tangible goal—a finished quilt square—cannot be underestimated. It kept people "tuned in" and "turned on" when they might have lost interest in a difficult situation. The shared project gave our class a real feeling of community which was nurtured by all our communication.

Making quilt squares with a small class of ABE students served to strongly reinforce a feeling of unity among the students. Sewing is a superb vehicle for increased communication. In every class, there was a time for helping one another and a time for talking about anything and everything. We were constantly engaged in problem-solving: deciding which pattern to choose, deciding which colors to put where, deciding which piece to attach to which piece and how, deciding how to draw, how to cut, and so on. A student with strong skills in the area of visual relations would be there to help a student who was a skilled sewer, and vice-versa. This was a chance for the group as individuals to learn in a most non-threatening way what help they could ask of other members of the group.

When asked what she liked best about making quilt squares Sara said precisely what I had been feeling, "Sharing with the group... We talked about the same subject — the problem and how to cut and sew... We all have a problem. We laugh and talk, that's the best."

SOME CONCLUSIONS

The question always arises, what would I change if I were to do this again or what advice would I give to someone who wants to do a similar project with another group of adult learners? I feel satisfied that my students gained a good deal of both mathematical and practical knowledge and a lot of pleasure and success from the experience of making quilt squares. The group I chose to work with had already established themselves as a cohesive unit, making quilt squares and sharing problems only served to strengthen their existing bond. However, if I had chosen a different group, one that was newly formed, or one that just hadn't pulled together, my project would have been the adhesive to help make a bond. The spirit of group, of the class as a team, can be a critical factor in the success of a math class. Group dynamics are greatly enhanced when the group has a common project or goal.

So what would I change? Most of the changes I would make are technical.

So what would I change? Where would I do more? What questions are unanswered? Most of the changes I would make are technical. I made a decision early in my project not to bring in a recorder and tape class conversations. I think this was a mistake, and I wish I had more of students' exact words, not only my memory of their words. When I sat with each student and asked specific questions about what they felt and what they had learned, I learned more than I ever expected. I wish I had talked individually to each student away from the quilt squares at least once or twice more during the course of our time together. I chose not to have the students keep a math journal during the project. Again, I wish I had decided otherwise. I think student journals would give valuable feedback to any teacher—a written commentary on student learning. I also think that whatever mathematics the students learn would only be reinforced when they write about it. I might have learned sooner why Sara hated sewing if I had asked her to keep a journal!

A long-term project, while providing class unity, is also just what its name implies: something which requires a long time and a commitment to putting in the necessary time to achieve the desired outcome without taking time away from everything else in the curriculum. In addition, it requires a lot of attention to planning curriculum around the project. We often felt rushed, and because of sporadic attendance and technical problems (sewing machine on the blink), some of the squares are not yet finished, though almost. If I were to do a similar project in the future, I would take more advance time to better prepare curriculum. I feel that this research project will serve as a kind of pilot project for me — a place to start from. I would love to do more quilting with other math classes.

Overall, I feel very happy about our quilt squares. I know the students enjoyed the time spent making them, even the time when they were stuck or frustrated. Students in other classes were envious of our beautiful work, and asked if they could do the same thing in their classes. I think the potential of learning language would make a quilting project valuable in an ESL curriculum (math or not math). The students and I discussed making quilt squares that told stories as well as squares with geometric shapes. Most of all, I agree with Sara that the best part of the entire project was the sharing. We spent hours sewing and talking to one another. I learned much more about the students than I would have if we had been doing the same old math stuff! I would not want to trade this experience for passing any test.

I hope that the insights I gained from our quilting experience will make me a more thoughtful teacher. I will definitely plan to give other math classes a shared goal to help unify the group. I want to give other math groups even more opportunity to write math journals. I hope that reading these journals will give me helpful information about students math feelings and math styles. I hope to explore the connection between geometry and art further with my math classes (string drawings and tessellations are examples of possible future ventures).

A fellow member of the math team questioned why it was so important for me to show off my students' quilt squares. I had to give this some thought. She was right, I have been showing everyone our class project. Why do I think the quilt squares are so wonderful? I have a great feeling of pride of accomplishment, for myself, and especially for my students. As Sara said, "I like it...it is beautiful!" The quilt squares are truly beautiful — their patterns, their colors are works of art. The time and caring that went into their creation are tangible to the five people who made them, and to anyone else who looks at them.

WHEN STUDENTS WORK IN TEAMS

By Barbara Blake Goodridge, Lowell Adult Education

What will happen if I break my math class into teams who will regularly attempt math activities and problem solving together?

MAKING A CHANGE

The adult learner is different from the same learner as a child. The child had a clean slate, an open mind, and few preconceived ideas. The adult who enters the classroom has formed attitudes about the world, education, and employment; but most importantly, about him or herself as a learner. In no subject is this more an issue than in math!

I asked my GED students, "How do you feel about math?" Their answers ranged from total dread based on fear and frustration to enjoyment and feeling empowered. The following responses are typical of the general adult population, I think.

Kimberly: I don't like math. I don't understand it, and if I don't get it I get aggravated.

Marsha: I like it. I think it's fun. I enjoy it.

Mary: It's hard, but once you get it I feel that it's a lot better.

Chris: I'm just no good at math. I think I'm not doing well, but I won't give up

George: It's difficult. Yeah, I'm having a hard time, I haven't done these in so long...too long.

The challenge that I face every year is how to establish an atmosphere in which people with different experiences and attitudes find room to explore and discover math skills.

Lowell Adult Education, where I teach, is a very large program. We have the luxury of having four different math classes meet at the same time, so that students can be placed in classes based on their math abilities. My GED math class met every Tuesday and Thursday evening from 7:55 to 8:45. The GED level

> ***"Much of the mathematics adults encounter everyday demands interaction between two or more people... it is essential that the mathematics curriculum of the adult basic education classroom involve strategies which promote skill development in shared problem-solving experiences and communication of mathematical ideas."***
>
> MASSACHUSETTS ADULT BASIC EDUCATION MATH STANDARDS, STANDARD 2: MATH AS COMMUNICATION

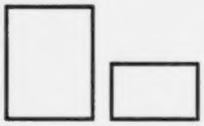
classes average 20-25 students. Due to the class size and time constraints, my approach to teaching a GED math class had always been rather traditional. I would begin the class by posing a problem and seeking suggestions on how to solve it. I would try to lead the student to the reasoning process and vocabulary that I thought they should use. I would conclude with a rule or process which I felt was "the best way" to solve the problem I would then give an assignment and walk around the classroom checking to see if each student was clear on the method. I was satisfied that the atmosphere was friendly and the students were given room to ask questions.

At the "Math Summit" in February, 1992, I learned of the new *NCTM Curriculum and Evaluation Standards for School Mathematics* and joined the group of teachers who met monthly to discuss how the standards might apply to ABE/GED math classes. Through those meetings and some reading, I began to have an interest in changing my approach to teaching math.

I do not make changes rapidly. Last fall, I tried a few isolated activities or approaches which were different for me with both my small individualized ABE math class and my larger, traditional GED math class. In some cases, the experiences were delightful and rewarding. In others, I learned what *not* to do the next time.

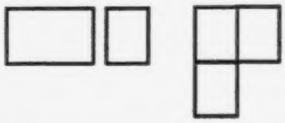
I do not make changes rapidly.

In February of 1993, I had given the GED class a worksheet that included adding and subtracting fractions with different denominators to determine if they knew how to work with fractions. Many of them did. For the benefit of those who did not and to demonstrate more visually to the others, I had the class divide themselves into groups and I gave them a "simple assignment". I gave each group one rectangle and another that was half its size.



I asked them to give me 3/4 of a rectangle back.

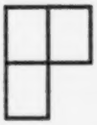
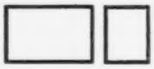
I expected everyone to fold the larger rectangle in fourths and give me one of the fourths along with the half. That would leave every one with 3/4.



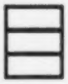
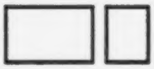
I assumed that they knew that 1/2 = 2/4. I would point out that by folding the 1 into fourths they had changed 1 to 4/4. Then I would show them the traditional way to write the problem.

$$\begin{array}{lcr} 1\text{-}1/2 & = & 6/4 \\ -\,3/4 & & -\,3/4 \\ \hline & & 3/4 \end{array}$$

Most of the students were more clever than I. They ignored the 1/2 of a rectangle and gave me 3/4 of the larger rectangle.

They gave me  with  left.

One group folded the fourths in strips.

They gave me  with  left.

I had to change what I had planned to do. Instead, I asked them what they had left. Each group had difficulty telling me. Some realized that the smaller piece was 1/4 but could not figure out how to add that to 1/2. One student said, "It's three quarters left, because one half plus one quarter is three quarters." When I asked her to show me how 1/2 + 1/4 = 3/4 she was stumped.

I learned a lot that evening about what the students did and did not know. I had some pleasant surprises. The students forced me to look at the problem differently. I realized that the class was much more involved with the assignment than if I had given them rules for subtraction on the board and a worksheet of problems.

They said that they liked the activity. I could see that they liked the groups. I also felt that I got closer to the reasoning processes of the students. How can I teach them if I don't know how they think?

I also felt that I got closer to the reasoning processes of the students. How can I teach them if I don't know how they think?

As I chose a math project, I had two goals. I wanted to continue to get closer to the thought processes or *reasoning* of my students so that I could see their *problem-solving* approaches. Secondly, I wanted to establish an atmosphere for them which would not negate who they are or what they bring to class. I wanted to establish a safe, stimulating atmosphere in which they could *communicate.* With this in mind, I formulated the question:What will happen if I break my math class into teams who will regularly attempt math activities and problem solving together?

METHOD AND FINDINGS

On March 2, I talked to the class about my idea of having them work in groups. I explained I would tape the groups upon occasion so that I could listen to how they approach a problem and get more of an understanding of the various approaches to problem solving. I said that this would be helpful to me as a teacher, but I also talked with them about the real life applications of cooperative learning experiences. I pointed out that the ability to work with others and communicate ideas was becoming more and more important in many occupational fields. The students were agreeable to the idea. No one expressed any reservations.

I considered how to divide the class into groups. Most of these students had been together since September. They had already struck up some friendships or acquaintances. For this reason, I decided to let them choose their own group. I feel now that that may have been a mistake. People who interact socially together do not necessarily work on a problem well together. The groups had some imbalances of math strength and assertiveness that became an issue.

During the next 2 months or 14 classes, I planned most lessons to include group time. At the end of that period I asked the participants to fill out a questionnaire about the pros and cons of the groups as they experienced them.

EXPERIENCES WITH GROUP DYNAMICS AND COMMUNICATION

I had planned to give a presentation of new material on multiplication of fractions to the whole class and then have them break into groups for the last half of the class. When I walked into classroom with a box of tape recorders, the students immediately started putting their desks into groups. There were twelve students. They formed four groups. I decided to take advantage of their eagerness. I gave them four discussion questions in order to get to know each other's plan and attitudes toward math.

Take a few minutes to talk about being a group. Each member of the group should take a minute (or more) to express:

✓ ***How you feel about math in general.***

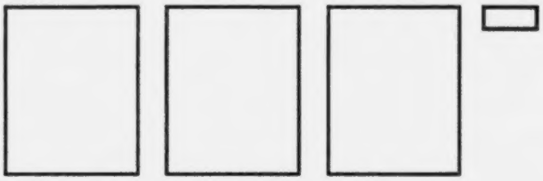
✓ ***What is your goal for yourself in math class this spring?***

✓ ***How long do you intend to continue attending classes?***

✓ ***How have you felt about doing activities in math, as well as worksheets?***

I taped these interactions. The room was buzzing. I was immediately pleased at their openness and involvement. Some of what they shared is quoted in the beginning of this article. After approximately 10-15 minutes of sharing, we reassembled as a whole class and I led an activity to present multiplication of fractions.

We ended the class in groups. I turned the tape recorders on. They were to demonstrate a subtraction problem in two ways. I asked them to do 3 1/16 - 2 5/8 traditionally on paper, but also to show me how they would take 3 1/16 rectangles and give me back 2 5/8 of a rectangle.



Later that evening, I listened to all of the tapes. I heard their interactions and I also heard my interventions, good and bad! Certain things stood out to me that became themes of our work together during the following classes.

Doing a problem and explaining what was done involves different skills or levels of understanding.

The members of one group worked on the answer using pen and paper calculations. Jose and George found a common denominator but struggled over how to subtract 10/16 from 1/16. Rana is Cambodian; he speaks softly, but confidently and shows them about renaming 1 as 16/16.

Then Jose said, "I can do the job but when she asks me a question, it's difficult for me to explain, you know? I have a hard time with that cause I'm a little nervous person myself. I mean you understand how it's done, but to explain it.... Like the other day she said, 'Good, how did you get that, Jose?' Gee, I don't know, I just came up with the answer." Rana answered, "Yes, almost everybody has a hard time to explain."

I heard these comments as I listened to the tape and I thought. If they really understood they would find it easier to explain. I listened to the tape of another group who had said something similar during class. During class when I checked their group, they gave me the wrong answer, but said that they had it a minute before. I assumed that they had had it wrong a minute before and were just now realizing it, BUT as I listened to the tape I heard that Karen and Mary had quickly solved the problem. Their problem was in giving an explanation of what they had done! This issue of communication jumped out at me. What are the factors? True understanding? Confidence? Vocabulary?

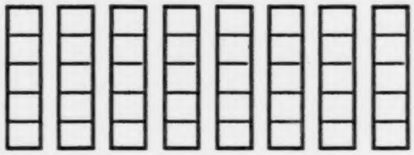
During a future class I heard Jose's voice on the tape say, "Yeah, but wait a minute, she is going to ask how we got it." So they discussed their methods. Then I heard Jose's voice playfully addressing me in the tape recorder. "Is that good enough for you, Barbara?" Students became aware that they would be asked to explain their process, so formulating an explanation became another step.

Students became aware that they would be asked to explain their process, so formulating an explanation became another step.

Group dynamic can be contagious.

In Group A, Marsha somewhat monopolized the process. Cindy assumed she was correct and followed her lead. Mavis, a young Haitian woman, tried to explain a different way (perhaps she saw Marsha's mistake) but Cindy and Marsha kept talking to each other and Mavis's voice faded away. When I came to see how they were doing I asked each of them questions and Mavis re-emerged briefly, but she is drowned out again after I leave. When Cindy and Marsha were satisfied that they had accomplished the task they called me over to show it to me. As they waited for me, Cindy said, "There we are all done, but you explain it to her." Marsha answered, "We will both explain it." (NOTE: Marsha says both acknowledging that she and Cindy have worked on this, but what has become of Mavis?)

They called me over and gave me 8 strips folded into fifths and told me that it was 5/8.



I asked, "If I take 8 strips folded into fifths I will have folded it into how many sections?" Cindy said 30. I looked at Mavis, and she said 40. Marsha then realized that it was 40 and said, "Is that right?" Cindy says, "No, that's wrong."

I explained that each strip was an eighth and asked how many should they have given me. They seemed to understand at this time that it should have been 5. We turned off the tape and as we cleaned up Marsha said, "I enjoyed this." Cindy said, "You learn more this way." As I listened to tape afterwards, I realized that I was only interacting with Cindy and Marsha by the end of the process. I too had let Mavis fade away.

Before the next class, I saw Mavis in the hallway before class. I told her that when I listened to the tape I heard her trying to help her group. She smiled but didn't say anything.

So this first evening of taping and groups had been a wonderful opportunity for me to "get closer to their reasoning" which had been one of my goals. Although, I was aware that the learners had several confusions and unanswered questions, I also could see that they had enjoyed the opportunity to work together. Even the groups who did not solve the problem alone said that they "learned more this way."

ISSUES WITH CONFIDENCE (SOCIAL AND MATHEMATICAL)

The students had seemed very enthusiastic about being in groups, so I planned for this entire class to be within the groups. I set up the tape recorders before the students arrived and made arrangements to include the new students and those who had been absent. There were 14 students present.

I gave each group the following instructions:

1. Take a few minutes to check in with your group members. The people in your group tonight will be your regular group members. Each person, please:

a. Give your name.

b. Tell if you thought the homework was easy or difficult.

2. Check the answers to your homework together. If anyone has a different answer to a problem, work it out together until all members of your group agree on the same answer.

When each member of the group agrees to the answers, put your names on your papers, collect the papers and turn them in.

Rule 1: You may not turn in your papers until all members agree to the same answers and understand them!

Rule 2: You may not ask Barbara for help, or to setle a disagreement unless all members of the group agree to.

The rules worked very well. Several students would laugh a little as they would ask, "Okay, now does everyone agree?" but they stuck to them faithfully.

Speaking up requires mathematical confidence.

In Group A the issue of confidence appeared with Chris. They started the ratio problems. Lisa, Cindy, and Marsha did all of the talking. They corrected each other and explained their reasoning well. At one point they disagreed, then settled on an answer. Marsha said, "Should we ask Barbara?" Cindy quickly answers, "No" I think that she wanted the satisfaction of working it through as a group. When I came to check their answers on the first few problems, they were correct, but they had not simplified the first one. I said, "On the first answer you got 15/36 which is correct, but you can reduce it." Chris said, "I got 5/12 is that right?" Marsha was irritated. "Why didn't you tell us?" Chris answered, "I didn't say anything because I don't know if it's right and I don't want to feel like a dumb ass."

It is interesting to me that she had enough confidence to write down the answer, but not enough to talk with the group members about it. Chris told me during the next class that she proudly told her boyfriend that she had something right when the others didn't and she and I talked about trusting her reasoning skills.

During a meeting with my math group members, I raised the question, "Can I justify math class time for dealing with group issues such as inclusion and self-confidence?" Esther Leonelli made an excellent point. Many times students tell us that they wrote the correct answer but changed it. Developing confidence in one's knowledge and ability is a test-taking skill, as well as a life skill.

Speaking up requires social confidence.

On the tape of Group D, I heard mistakes as Jose and Grace discussed a problem. Bo, another quiet Cambodian man, tries to tell them. I hear his voice on the tape, but they didn't acknowledge him. He tried to interject twice while they were working through the problem. Finally, as they finished with the wrong answer, Jose asked, "So do we all agree?" Quietly, but with confidence, Bo said, "No" and showed them their error. This was very satisfying to me as I listened to the tape.

Over-confidence hinders listening skills.

The students were not the only ones who had issues with not listening! When I typed the problems, I had wanted to include a problem with subtracting from a whole number. I expected a few students to still be having difficulty with this. However, I made a mistake in phrasing the problem.

A 2-quart pitcher is full of iced tea. If 3 people each take 3/5 of a pitcher, how much of a pitcher is left?

Several students tried to tell me that the problem could not be done. I assumed that their problem was borrowing from a whole number. I kept encouraging them to "draw a picture." Finally, to make my point, I read the problem aloud to Group E and realized my mistake. The problems should have read "*Two* quart pitchers." As penance I had to listen to myself five times on the tapes, going from group to group and explaining my error.

A recipe calls for 1-1/2 cups of sugar. I have 3/4 cup in the canister and 2/3 cup in the sugar bowl. Do I have enough sugar for the recipe? Yes or No

In Group E they also agreed that there was not enough sugar, but one student said, "Yeah, but you know what, I would make the recipe anyway because 1/12 is all that they are missing and that isn't very much." Good practical reasoning! Perhaps this is an example of someone who has confidence socially and mathematically. He had enough confidence to go beyond the "right answer" to using common sense (and number sense).

READING IS KEY

We continued doing ratio and proportion by reading explanatory material and working on the exercises in the groups. I don't think that this worked well. They didn't really read it. They skimmed it and got to the exercises too quickly and unprepared. The students did take the materials home and some of them read it more carefully at home, but too many of them did not work on it independently.

One of the activities which we did was a cooperative learning exercise taken from the Equals Project's *GET IT TOGETHER*. Each student in the group is given one or two of six clues. They each take a turn reading their clue(s) aloud and then work together on the solution. The person who has a particular clue is responsible to see that it is not overlooked.

MGB MPG

Emily Knobloch drives a white MGB. Every fortnight she puts nine gallons of gasoline in her tank to fill it up. ***Emily is planning a 100-mile trip. How much will the gasoline cost?***	***Where Emily lives, gas for her car costs $1.08 per gallon.*** ***An MGB is a British-made sports car. Emily's MGB handles really well, bt it's often in the shop.***
Ordinarily, the only driving Emily does during the week is her trip to work and back, which she does five times a week. That round trip is 15 miles. ***A fortnight is two weeks, by the way. Fourteen days.***	***Emily is very unusual in that she drives fewer than 5,000 miles per year. but how much will gas cost for her trip? It might help to figure out how many miles per gallon (MPG) Emily's car gets.***

ABOVE ARE FOUR OF THE SIX CLUES IN THE COOPERATIVE PROBLEM FROM GET IT TOGETHER, **BY TOM ERIKSON, EQUALS PROJECT, LAWRENCE HALL OF SCIENCE, BERKELEY, CA, 1989.**

Two things stood out very strongly during this activity.

Reading is key and the students don't realize it.

The problem talked about a "fortnight." I heard several students read it as "four nights" or the "fourth night." It wasn't until they read the clue that a fortnight is two weeks that they admitted or noticed that the word was "fortnight" I think that they might slip over what they don't know and hope to slide by. I think that most of them don't attempt to find out what something new is or means.

As soon as they have a few numbers they are in a hurry to use some operation to solve the problem.

It seems the students think, "There are numbers... use them... solve." One group told me it would cost $5,288 to drive 100 miles! They read the problem superficially! As I walked around from group to group I overheard comments like, "Don't you think that we should divide something?" or "This is definitely multiplication, what should we multiply first?"

Everyone was very engaged with the problem. Each group solved the problem using different methods. I asked everyone to write their solutions on the board. Groups A and E had solved it by a series of multiplications and divisions.

Marsha went up alone from Group A. The rest were working on the homework problems. I asked Chris if she wanted to check on Marsha. She laughed, "No, I think she can handle it."

Pete, Maura and Emma (the members of Group E for the evening) went to the board. Pete wrote, while Maura and Emma made suggestions. They showed and labeled the parts of the problem that they had used and even the parts that they did not use (9 x $1.08 = $9.72). I asked about that figure. Pete said, "Well, we tried that but couldn't use it."

In Group D, Jose and Rana had solved it by writing different proportions. I asked Jose and Rana to each write their solutions.

Jose wrote $\frac{1.08}{18} = \frac{x}{100}$

Rana wrote $\frac{\$9.72}{162} = \frac{x}{100}$

I had everyone sit together and look at the solutions. Pete explained theirs, and Marsha said that theirs was the same. Cynthia pointed to Jose's solution and said, "Look at this one. It has a proportion. That looks so much easier than the way we did it." I emphasized that there were several paths to the right answer and proportion can be a tool. I told everyone to use the method that makes the most sense to them. I had not noticed Rana's proportion tucked up on top of the board. Pete interrupted me, "I want to hear about that one. They used the $9.72." We looked at Rana's proportion and discussed the units. Pete said, "See I was right; we could have used that!"

Cooperative learning was an excellent experience for everyone. I could see such satisfaction in their expressions. I continue to try different approaches and activities. Some were more successful than others. Whether or not I felt good about the evening—the students always seemed pleased. They liked the groups! Some of their comments after the first five classes were, "You learn more." and "If you don't understand and someone else does, they can explain it to you." " If we didn't like it (group work) you would know!" When a new student joined our class, I explained that we were working in groups "most nights." Cindy corrected me, "We work in groups every night now; that's how we like it."

When a new student joined our class, I explained that we were working in groups "most nights." Cindy corrected me, "We work in groups every night now; that's how we like it."

These weeks were delightful for me. I continued to enjoy the opportunity to hear how the student approached a problem rather than have them listen to my approach. I also saw them taking an interest in helping each other.

CONCLUSION

What will happen if I break my math class into teams who will regularly attempt math activities and problem solving together?

We will all teach each other! I hope I have successfully conveyed to the reader what a wonderful learning experience this has been for me, the teacher, as well as for the students.

Kimberly: I think it's easier to learn when you're one person in a group of 3 or 4 people, than it is to be one person in a group of 20 or 30 people. I think that was one of my problems in school.

***Cindy:* You get more than one idea on how to do the problem.**

***Marsha:* We all work together and it is a lot of fun. I feel we learn more in groups because we all figure out the answer together.**

I found the tapes invaluable. Obviously, I can't be in four places at once. The tapes helped me to hear what I had missed and, in some instances, they corrected my perceptions. For example, I heard on the tape that the people who contributed the most in the groups were not necessarily the ones who would speak for them when they reported their process. Next year, I will consider having the students listen to their own tapes during a future class. It would provide opportunities for the students to address group issues as well as problem-solving approaches.

In my classes next year, I will definitely have group activities within my class, I will be careful in choosing the initial activities. Few adults have had opportunity for cooperative learning experiences in our highly competitive society. I learned that if I wanted students to work together, I needed to establish the proper setting and atmosphere for that to happen. It is very important to provide initial exercises which are within the student's range of understanding and which are entertaining. As the students become comfortable with some non-threatening problems, we can expand to more challenging ones.

The principal area in which I want to grow next year is in facilitating and summarizing the groups' learning. I will heed my students advice and give them more structure in introducing new material. I think the group activities can be utilized for discovery, practice and problem-solving, but the whole class setting may be better for introduction, definition, and summary.

LEARNING BY DOING, OR WHAT HAPPENS WHEN STUDENTS WRITE THEIR OWN MATH WORD PROBLEMS?

By Tricia Donovan, Franklin/Hampshire Employment and Training Consortium

Problem: I'm hungry because there was no food at home, but someone has just paid me $5 for Avon products. I get an Egg McMuffin. How many egg and English muffin breakfasts could be made at home for the price of the fast food?

Problem: The landlord told the tenants in our complex that rent will no longer include electricity or heat. What will that end up costing me over the next year?

Problem: I want to make curtains for my windows, and I know I need to start saving money for the material. How much fabric will I need for two windows? What will it cost? When will I be able to afford the material?

When we are deeply involved in the struggle to solve a mathematical problem, my GED students often ask, "when will we ever use this?" The evidence from their own lives (above) drives home the point: you use it everyday. Mathematics is not esoteric academics, it is the real thing — a way to determine proper responses to or predict actions within real problems.

The disassociation with math expressed by my students made me wonder what it was about the way math is taught that makes people feel it is not real. The GED math books are full of purported "real-life" problems, so why did my students always wonder what the purpose of learning "all this math stuff" was? And why, if they had read math terms for years, did they verbally stumble when trying to express their questions regarding math word problems? Finally, why did I feel like I was imposing problems on my students in math class, despite concerted efforts to manage a student-responsive classroom?

> ***"In the adult basic education classroom, curriculum design must include approaches to teaching mathematics as communication which allow the learner to appreciate the value of mathematical language and notation in relation to mathematical ideas."***
>
> **MASSACHUSETTS ADULT BASIC EDUCATION MATH STANDARDS, STANDARD 2: MATHEMATICS AS COMMUNICATION**

Because the development of self-confidence through acquisition of academic skills in a non-threatening environment is a teaching priority for me, and because I want all my students to feel empowered and liberated within their rapidly advancing technological culture here in the United States, it is important to me that they become successful, capable problem solvers. To be good problem solvers, I believe they need to be bi-lingual: both literate and numerate.

Was there some way to integrate the two languages of numbers and words in a context that emphasized the real and personal natures of mathematics?

Yet though I was partially developing numeracy and literacy in my Monday-Friday, 9 a.m.-12 p.m. classes, I wondered if I could do a better job. Was there some way to integrate the two languages of numbers and words in a context that emphasized the real and personal natures of mathematics?

With these questions and pre-existing goals in mind, I thought (in February 1993) that if I was going to research any math standards for the Massachusetts ABE Math Standards Project, that research would be centered on problem solving and communication. The thought generated the question:

"What happens when students write their own math word problems?"

This simple question would serve me in many ways. For years, I have maintained that I don't teach subjects, I teach students. Having students create their own math word problems would allow for a student-centered curriculum which I feel is imperative, and which is also supported by the position of the *NCTM Curriculum and Evaluation Standards for School Mathematics* that "knowing mathematics is doing mathematics." *(p.7)*

In addition, I would be addressing two of what I consider the most important life-long learning standards: Problem Solving and Math as Communication. As citizens, as parents, as co-workers we all need to solve problems and relate to others our reasoning processes and solution strategies. In the words of the *NCTM Standards*, "Problem solving must be the focus of school mathematics" and "as students communicate their ideas, they learn to clarify ... their thinking."

To facilitate my research, I bought each student a five-subject notebook which we divided into sections: my math word problems and solutions; others' solutions to my problems; others' problems and my solutions; math journal and math vocabulary list.

To facilitate my research, I bought each student a five-subject notebook which we divided into sections: my math word problems and solutions; others' solutions to my problems; others' problems and my solutions; math journal and math vocabulary list.

Preliminary discussions with other instructors and my own reflections made me conscious of the need to: 1) track strategies for solving problems; 2) acquire knowledge of mathematical terms; and 3) review the individual learning processes and experiences. I hoped the notebooks would allow students to fulfill these needs.

The notebooks also imparted a sense of personal involvement with the research project. Everyone would have a private record of her/his math education process to carry with them after graduation. Somehow, the notebooks also made the concept of the project more real, so once they were in hand, we felt ready to embark together on the research journey.

TIME TO GROW SOME MORE

Though I had, in many ways, emphasized problem solving and communication skills in my math classes, I had never asked students to regularly write their own word problems as a means of practicing these skills. Now, within the framework of the research project, I would have about 16 class sessions to do so.

During the research project period, class followed the regular class schedule which students designed earlier in the year. The important difference lay not in logistical format, it lay in educational format. Now we would cease to focus on manufactured (textbook, teacher) problems. Our class time, which was almost always less than the scheduled 2-1/2 hours, due to car breakdowns, emotional upheavals, sick children, etc. and our normal routines of Monday morning check-in meetings and problem-of-the-week analysis, would be devoted to the articulation and solving of student-generated problems. So I thought...

I did find that the problem-of-the-week (P.O.W.) analysis intersected neatly with the research project. These open-ended or multiple solution questions presented each Monday and discussed the following week, were valuable entry points for discussion of problem-solving strategies. Each student could develop her/his own solution process (or small-group process) and by sharing it with others, broaden everyone's understanding. These problems also acted as problem prototypes from which students could develop their own math word problems.

For instance, when we worked in pairs to solve the P.O.W.:

I need five more $2 bills than $5 bills to make a purchase. How many of each bill do I need?

the class noted on the class journal sheet (a variation on the personal journal used to appease students tired of writing on their own which also reaffirmed earlier observations; much is learned by sharing thoughts & communication can sharpen thinking) that the ways to solve a problem of this type included:

✔ "Using our own brain cells"

✔ "Using money denominations"

✔ Knowing "the difference between the # of bills and the amount they represent"

The P.O.W. class journal review of the process also revealed these insights about learning:

✔ "Hands on experience is easier to learn"

✔ "Everyone has different ideas and opinions"

✔ "Have to learn to compromise."

After we worked on solving the P.O.W., we devoted a second day to creating student-generated problems of a similar nature. Students worked in small groups to write their problems, but since we were logging everyone's work, each person was able to write a problem that reflected their own level of learning and learning style. One student figured out the relationship between $2 and

$5 bills in the P.O.W. and was subsequently able to generate many problems written in the same format. Another student had seen the relationship to multiples of a number, whether it ended in zero or five for instance, and devoted his time creating verbally complex problems.

Regardless of their level of understanding though, it was clear from their concerted effort and conversations that the writing of their own problems "brought it home" for them. As they worked and re-worked their problems (sometimes after someone else pointed out they couldn't be solved as written), they confidently showed each other how they arrived at their answers, often using the money denominations we'd created to prove their case.

I was beginning to feel this research project just might work out. It was important to me that they continue to learn math concepts they'd need for their GED tests, too, so I always emphasized that the techniques for problem solving that they were acquiring could also be applied to test problems. If time allowed, I might present a test problem for them to solve using real objects or pictures, for example, after we'd learned that strategy.

Still, the continuity required for what I considered traditional research was not obtainable in the GED program. Four of the 12 students who started the research project graduated within six weeks, another found a job; four more started class. I soon realized I'd be gathering snapshots of student work rather than making a movie of it. Whatever I was to learn from the research would be winnowed from scattered experiences that revealed similar results.

I soon realized I'd be gathering snapshots of student work rather than making a movie of it.

In addition to the P.O.W. snapshot, there are three others that I mentally examine over and over. They involve:

✔ The GED Test Exploration exercise

✔ Measurement Exploration

✔ Math in Your Life exercises.

From these snapshots, I piece together the bigger picture of what happened when students regularly wrote their own math work problems.

The GED Test Exploration exercise was not presented first in my research process, though it might have been, for through this detailed pre-test examination students' sense of context was established. It was necessary to underscore the personally relevant reason we study problem solving in GED class. By picking apart the questions and answers on a real pre-test, the students concluded that if you want to pass the GED math exam, you better know how to read, understand, represent and solve math word problems.

As they worked in small groups on the various pretests, questions tumbled forth endlessly:

"What's a geometric shape?"

"What's a single-term solution?"

"What's a solution set-up?"

"Is a graph a visual representation?"

Math terms were being defined left and right, mostly through further questions — "What does single mean? Term? Solution?", or discussion as people bandied ideas back and forth.

The language of mathematics was becoming real to students as it became necessary to complete a task in which they were personally involved. Still, I had to constantly remind everyone to add the new terms they'd learned to their vocabulary lists. Study skills need reinforcement as do organizational skills. Most students wrote new words on whatever paper lay in front of them at the time.

When students reached the section where they were supposed to solve some of the problems (division of this work was a fascinating study in group dynamics), everyone was anxious to begin work on the problems they'd reviewed. In fact, several students completed all the problems, a situation I encouraged as a means of checking each other's solutions. Meanwhile, self-confidence was growing as fast as the vocabulary lists. I heard students say, "I think I can pass this test," though days before they might have said, "Math is going to be my last test; I have a lot of work to do." Even students who hadn't yet grasped the basics were chomping at the bit. For once, I was restraining people rather than pushing them.

Despite the euphoria about several stimulated classes, I wondered if this activity really fell within the parameters of the research question. After all, students weren't actually writing their own questions. But they were certainly asking them, I thought.

And at the end of the project, when students evaluated the various lessons, they ranked the test exploration as the most helpful/useful, most interesting lesson from which they learned the most about math and themselves.

I guess serendipity is part of research.

MEASUREMENT EXPLORATION

Our Measurement Exploration exercises formed another of my favorite research snapshots and more directly fell within the project parameters. The picture began to unfold spontaneously from Louisa's Monday morning report of her weekend's activities. Louisa reported to the class that she and her family had made maple syrup that weekend and had boiled down enough sap (30 gallons) to make almost a full gallon of syrup. The mere mention of a mathy word like gallon always provokes a question from me, and did now.

The question I asked was, "What do you mean 'almost' a gallon?" I strove here for language precision because it is vital in all communication, including mathematics. Also, how can we, as the *Standards* advise, develop common understandings of mathematical ideas if we don't establish a common vocabulary of concepts. Louisa responded, "Well, it was a little less."

Me: "Well, how much do you think you made?"

At this point, others in the class engaged and began to call out fraction amounts — "1/2 gallon; 1/3 gallon; 3/4; 2/3." I gave no indication whether answers were close or not. I did offer a guess, "9/10." Students asked quizzically, "9/10?" I explained that this fraction to me meant almost a whole (gallon). Louisa said she didn't know how much of a gallon she'd made; she only knew she'd made 15 cups of syrup.

As usual, the desire to keep class momentum superseded the desire to work though a step-by-step analysis of the problem-writing process. I chose, mostly from habit, to model the writing of a simple problem rather than work arduously with students to create one. So, I wrote:

"Louisa made 15 cups of syrup. What part of a gallon did she make?"

The class was stumped. A brief, rare moment of silence ensued. I waited a few seconds then asked, "What do we need to know?"

Someone answered, "How many cups are in a gallon."

Again, I could have stopped to discuss, in general, how we know what it is we need to know to solve a problem, and in particular, how knowing the number of cups that are in a gallon could help us solve the stated problem. (Analysis of my class journal really does help me see better what happened in class as well as how to improve my processes!) But I didn't stop; I succumbed to habit and time pressures.

I started firing directive questions regarding ounces and cups at the students to lead them to figure out that if there were 64 ounces in a gallon and 8 ounces in a cup then there were 16 cups in a gallon. So, Louisa made 15/16 of a gallon, they concluded from their answers to my questions.

I noted how they turned to each other for answers and how they trusted certain classmate's responses and how that sharing and trust generated greater confidence. Tentative answers mumbled off to the side became face-front, clearly articulated assertions.

Once again, I witnessed the development of students' confidence in their ability to solve problems. Also, once again, I was not on task for the research project. But as I reflected on this engagement of minds, I understood that the seeds for a future, on-task assignment were present. What we had begun to do that day was to explore equivalents in measurement. Ah-ha!

About two weeks later, we began our Measurement Exploration. We generated a long list of measurement terms — how our math vocabulary was expanding — and worked as a group to define those words, often by their equivalencies. We expanded the list from units of measurement, such as inch, mile, etc. to concepts of measurement, such as area, volume, conversion.

Not everyone understood all the terms, but in a diverse group like our GED class, that's a given. What was important was that everyone knew at least some of the terms and was aware that other terms existed, and that everyone could participate by utilizing their personal knowledge bank.

Students were asked to follow-up the listing exercise by creating their own math word problems. Some jumped right in and wrote simple problems, more to get it done than to explore math, I assumed. Still, it seemed a success for them to be able to write something down, to get it right and not appear foolish, so I did not push them to do more. Later, however, at least one of those students noted that "other people's problems seemed more interesting" and "I wished I'd written more". (Reflection time, whether verbal or in writing, proved essential in assessing what we or others had done, I realized. I also realized that the length of time required to finish what we were doing often precluded journal writing. Verbal reflection had to suffice.)

Other students wondered what they should do, so I suggested they try writing a problem related to area or volume. Still others drew directly from their own experiences to create measurement word problems. All were becoming acclimated to the problem-writing process in their own way. They were learning by doing, including entering their problems on their computer disks. They were becoming mathematicians practicing the art of authoring problems.

They were becoming mathematicians practicing the art of authoring problems.

As developing mathematicians, students also needed to discover and acquire problem-solving strategies. Whenever we wrote problems in class, we also solved them using whatever methods seemed to work. For instance, a multi-step area problem solution required students to "break it down" into manageable steps, though not everyone might recognize the need to do that or that that was what they'd done. The post-solution discussions within the larger class group became lively moments of discovery and sharing, as problem-solving processes were identified and named.

We kept a running list of strategies (tools) we picked up as we progressed. Gradually, as we experienced different math and problem stories, we accumulated the following strategy-tools:

- ✔ Use real things
- ✔ Draw pictures
- ✔ Draw charts
- ✔ Separate out the facts and list them
- ✔ Break it (the problem) down
- ✔ Look for multiples to shorten addition problems
- ✔ Eliminate unnecessary information
- ✔ Narrow your options (includes estimation)
- ✔ Re-write the problem
- ✔ Substitution

Clearly, the more times we repeat math terms and methods, the more numerate we become. As math is connected to a broader range of personal experience, and so reinforced continually, it becomes more habitual to think and communicate mathematically.

The "doing" of mathematics does impact students.

Rather than reacting to problems as victims, whether of a test, a teacher or life itself, the students in those classes acted as sources of knowledge. They had the power to *present* the situation rather than simply respond to it. Such empowerment is the cornerstone of future successes.

MATH IN YOUR LIFE

The final snapshot series — Math in Your Life — developed from my ongoing desire to help students discover the truth that math is real.

After hearing from a fellow teacher that her students genuinely enjoyed doing mental math problems, I decided to introduce that practice as a warm-up activity. I modelled the process: "I'm thinking of a number, and if you add two to it, you get 27" then "I'm thinking of a number, and if you multiply it by four and add two you get 102," and finally, "If I go to the store and purchase an item for 75 cents and I hand the clerk one dollar, how much change do I receive?" We'd then proceed around the table with everyone verbally presenting a mental math problem. (Sometimes students made errors, but we took mistakes in stride. We even applauded them as learning opportunities, asking, "Well, how can we make that work?")

Everyone fake-moaned when we would do "loco" math, as Luz termed it, but became involved in seconds. They liked creating math problems! Maybe it was just the writing part that stumped some students. So, one day when an observer was present, I decided to extend the exercise a bit by asking each student to first think of how they'd used math in their lives over the weekend. We then asked questions about each person's experience, and these questions became problems as I wrote the information on the board. Because this was all new to me, I like to rationalize, I asked quite a few directive questions to keep the process flowing. But everyone was engaged (I could tell because no small group conversations were heard or doodling observed) in figuring out how many miles per gallon Todd got in his car and how much steak Ginger had to cook to feed her family Sunday, etc. And some students actually began to phrase their own questions/problems that day.

"Let's do that again," students said after class. So, we did. Eventually, we extended the mental math information and problems into written math problems to graphically link the two languages —math and English. Thinking of ways to connect numbers into math sentences and number sentences into story problems was becoming a way of life in our class.

Now, more and more students were gaining the confidence to take and pass the math exam. We hadn't "covered" a full textbook curriculum, but we had stimulated and expanded our ability to understand mathematical expressions and utilize an array of problem-solving strategies.

TIME TO GO

Through the research process, I learned much. I learned to trust that by presenting a math curriculum attuned to the *NCTM Standards* the class gained. We had accomplished "group projects that required students to use available technology (calculators and computers) and to engage in cooperative learning and discussion" even as each was allowed to personally struggle and grow through development of individualized language to express math concepts via word problems. In addition, I better understand the importance of ownership in all work as students connected math to their own lives in their own words and numbers.

Practice and ownership had made the acquisition of concepts and work of writing problem texts more defined and accessible for the learners

Practice and ownership had made the acquisition of concepts and work of writing problem texts more defined and accessible for the learners, but it had also helped me better understand the ins and outs of word problems.

When the students evaluated the lessons and processes of the project, they noted that while much effort is required "to put a math problem together in words," "it's a fun way to learn for a change." The nervousness and fears many felt when attempting to write math word problems for the first time in their lives (only two of 16 had done it before), gave way to confidence as they "got the hang of it."

Genuine math comprehension seemed to develop as they acquired a greater math vocabulary, responded to problem writing stimuli and brainstormed and reflected upon problem solving strategies.

We all grew.

One of the best parts of all, for me, was that the math learning was achieved in a way that allowed each adult to participate successfully at an individual pace, even in a classroom setting. Each problem, like each lesson, became a snapshot of student progress towards the ultimate goal of competence and confidence in solving life's problems.

We actively multiplied, divided, found proportions and unknowns in a gentle, humorous environment where it was safe to acknowledge successes and failures, where the emphasis is/was on understanding math processes, so that each is confident of his/her answers. One may still rely on paper and pencil or chalk and chalkboard to verify computations, but confidence in one's reasoning is the push to pick up the pencil/chalk and figure out the problem.

The confidence that evolved from "doing" math was, and is, essential to each one's academic and personal progress. The ability to, as the *Standards* say, "formulate problems from situations within and outside mathematics" was completely reinforced by self-creation of problems and led to successes at the students' varied developmental levels. Moreover, though the change from passive to active learning, from having math done to them to doing it themselves, was as one student said, "harder than it appears," it was also more fruitful than I had hoped when we first opened our research notebooks and embarked on this learning journey.

ASSESSMENT

EVALUATING CREATIVE MATHEMATICAL THINKING AND THE GED

By Sally Spencer, The Care Center

My interest in the role evaluation plays in mathematics education began when, with the Massachusetts ABE Math Team, I helped to adapt the *NCTM Curriculum and Evaluation Standards for School Mathematics* to the adult classroom. From this experience, I came to believe:

"In the adult basic education classroom, methods and tasks for assessing students' learning should be aligned with the learner's and the curriculum's goals, objectives, and mathematical content."

MASSACHUSETTS ADULT BASIC EDUCATION MATH STANDARDS, STANDARD 12: ASSESSMENT AND EVALUATION

> ***Evaluation is a tool for implementing the Standards and effecting change systematically. The main purpose of evaluation, as described in these standards, is to help teachers better understand what students know and make meaningful instructional decisions. The focus is on what happens in the classroom as students and teachers interact. Therefore, these evaluation standards call for changes beyond the mere modification of tests (NCTM Standards, p. 189).***
>
> ***Instead, our instruments must reflect the scope and intent of our instructional program to have students solve problems, reason, and communicate. Furthermore, the instruments must enable the teacher to understand students' perceptions of mathematical ideas and processes and their ability to function in a mathematical context. At the same time, they must be sensitive enough to help teachers identify individual areas of difficulty in order to improve instruction (NCTM Standards, p. 192).***

Even though I agreed with the NCTM's new vision for assessment, I wasn't sure what that meant in *my* classroom at The Care Center in Holyoke MA, a city in economic decline (where over thirty percent of the residents and over seventy percent of the public school children are Hispanic). I was frustrated further as I confronted my own prejudices about evaluation. When I was a student, I believed that the purpose of tests was to prove my mastery of a subject. If I received an "A" or "B", I understood the material and if I received a "D" or "F", I didn't. However, since grades are not even an issue in the GED classroom how can adult students, many of whom harbor preconceptions similar to mine, measure their progress? How do they and their teachers know when they are ready to take their GED test? Does the GED test even meet any of the above goals of evaluation tools? And, if the GED doesn't serve these purposes, what other evaluation tools exist? Can I or my students better discern the mathematical understandings which they have or need to develop from any of these alternative tools? While I knew I couldn't answer all of these questions with just one project, I decided to join in the Massachusetts ABE Math Team's teacher research to try to address some of them.

I had never been convinced that a passing score on the GED test really indicated mathematical understanding, but I wanted to challenge my bias by comparing what I could learn about my students' mathematical knowledge using various evaluation tools including the GED. I did learn a lot about my students' mathematical skills and talents during the course of my research. In addition, I learned about asking questions both verbally and in written format and I learned how important it is to align the evaluation tools with the methods of mathematical exploration used in the classroom. Most importantly, I learned how limited the information is that one can get from written exams (my bias against the GED as a measure of mathematical understanding was valid). I learned that opportunities for a truer understanding of my students are available from think-aloud protocols.

By beginning my teacher-research project researching existing alternative assessment tools, I learned about think-aloud protocols, open-ended questions, problem sorting and other evaluation techniques.[1] I was drawn to open-ended problems because I felt that they more accurately reflected the real world problem solving which my students face daily. A math problem is "open" if either (a) it can be solved using more than one strategy, (b) it requires some interpretation or a value judgement, or (c) it has more than one correct answer. The Care Center's students, who are all young women 15-21 years old, either pregnant or parenting and receiving some sort

I was drawn to open-ended problems because I felt that they more accurately reflected the real world problem solving which my students face daily.

of public assistance, practice open-ended problem solving monthly as they stretch their welfare dollars to meet their family's basic needs. The choices they make as they strive to provide the best life for their children on a tight budget requires that the young mothers make value judgements. Because different students will make different choices, there will be several correct answers. It is the creative problem solving which is a part of every day life for these students that I wanted to build on to help them develop their mathematical power and to help me assess it.

I also believe that since the state has added open-ended problems to their standardized tests for the public schools, it is feasible that they could be added to the GED as well. However, because open problems allow for several correct answers, scoring is difficult. There are many types of scoring rubrics for open problems which then need to be tailored to each problem situation. However, if open-ended problems were to be added to the GED, I believe that a relatively simple, holistic method similar to the scoring used for the essay (an open-ended problem already in use on the GED) would be appropriate. Nonetheless, I became grateful that, because I was an adult educator, I did *not* need to worry about grades. This enabled me to use open-ended problems without having to devise a scoring rubric, even a relatively simple one, for each problem. After all, with my project I was focusing on understanding my students' mathematical strengths and weaknesses.

In addition to presenting open problems, I decided that I wanted to incorporate some form of self-evaluation in my assessment process. I wanted to know how accurately my students could identify their mathematical abilities and I was curious what role their feelings play in the testing process. I strongly believe that students, particularly The Care Center's population of disempowered young women, need to learn that they can turn to themselves as authorities. I hoped that by asking the students to describe their feelings throughout the testing process and having them choose which problem they would solve, they would begin to see themselves as having control of their academic growth. However, students have *their* agenda, too. Every one of my student's ultimate goal is to pass her GED (and not necessarily to become more mathematically literate). So I felt that it was equally important to tie this evaluation project to the GED test directly.

My question for the project became: "What can I learn about my students' mathematical understanding using various evaluation tools: questions from the multiple-choice GED practice test, a closed version of the selected GED test questions (no multiple choices), and an open-ended version with the goal of helping students determine their academic progress and as a tool to improve instruction?"

CONTENT AND PLANNING

At The Care Center, students choose whether they want to prepare for the GED in English or Spanish. Because the students are separated by language of choice and not by academic ability, in both the English and Spanish classrooms students reading at the third grade level study alongside students reading at the tenth, eleventh and even twelfth grade levels. There is also a broad range of mathematical ability among the students in both classrooms (I teach in the English class where frequently the bilingual students will explain ideas to each other or mumble calculations to themselves — as well as gossip, or "bochinche" — in Spanish). Therefore, I needed an instrument to evaluate diverse mathematical content.

As I began choosing problems from the various forms of the official GED Practice Tests (Form AA, Form BB, and Form DD, available from Steck-Vaugh Company), I decided that it would be appropriate to include a problem using whole number operations; another focusing on fractions, decimals or percents; one to test algebra; and the last addressing the geometry content area. I had a hard time choosing an "algebra" problem-type. Either I was able to find a problem in which algebra was one of many tools that could be used (for example, a distance problem), or the problem's algebra was so restrictive that I didn't see any way to adapt it for open-ended problem solving (for example, a question which asked the student to evaluate an algebraic expressions, given the values to substitute for its variables). I decided to eliminate the algebra content area and substituted a problem which used whole numbers in multi-step operations. With the four GED sample questions selected from the practice tests (#1 from Form BB p. 55 [Figure 1]; #15 [Figure 2] and #16 [Figure 3] from Form AA p. 57 & 58; and #20 [Figure 4] from Form DD p. 60), I then rewrote the problems two ways: as closed, but not multiple-choice, and open-ended problems.

Many of The Care Center's students have test anxiety, so I decided that I wanted to explore the process of evaluation, as well as the type of question.

I also began to think about how to administer the test. Many of The Care Center's students have test anxiety, so I decided that I wanted to explore how the process of evaluation, as well as the type of question, would impact the outcome. I decided to learn to administer think-aloud protocols, an assessment technique in which the student begins by solving a particular problem out loud and explains the thought process which she is using. The evaluator's job is not to instruct nor to indicate whether the student's answer is right or wrong but to try to discern the student's mathematical understandings and misunderstandings. Because the student's approach to the original problem posed in the think-aloud protocol may indicate the need to explore a particular mathematical concept more thoroughly, never getting "the answer" to the original question is immaterial. I felt that this process went to the heart of what evaluation is for: determining students' perceptions of mathematical ideas and processes by reasoning and communicating mathematically to solve problems. From this information, I hoped to be able to improve my instruction. Unfortunately, because I decided to compare the *process* of the testing as well as the *type* of test questions, I needed to write two more versions of each problem-type for the four selected GED test questions which I would use during the think-aloud protocol phase of evaluation (at the time it did not make sense to have the students solve multiple-choice problems aloud so I did not need to rewrite that type of question to use in the context of an interview, now I'm not so sure). Then I needed some way to incorporate the self-evaluation.

The end result was:

PHASE I. The students were told that they would be given a math test.

Step 1. The students were asked to review — and *not* solve - the four original GED test questions which I had selected, and rate their perceived ability to complete them successfully, as follows:

A= I can do this in my sleep

B= I'm pretty sure I can do this, but I'll have to think kind of hard

C= I can start this problem, but I'm not sure I'll be able to finish or I might not get the right answer

D= I have no idea what's going on.

Step 2. They were asked to solve one of the problems which they'd rated difficult, but solvable, B or C.

Step 3. The young women were given the "problem page" which matched the GED question they'd selected, and asked to rate their feelings about math, about math tests and about the particular problem they'd just done:

This is how I feel about math:

love it ———————————————— ***hate it***

This is how I feel about math tests:

nervous ———————————————— ***confident***

This is how I feel about this problem:

smart ———————————————— ***stupid***

Step 4. They solved the rewritten test question in both the closed and open formats "showing all work".

Step 5. They were asked to complete "This is how I feel now:" and were given a list of feeling words (gratefully lifted from a summer math camp where I worked before coming to The Care Center) to use for reference if they wanted:

ENTHUSIASTIC
INTENSE
UNINTERESTED
COOPERATIVE
ATTENTIVE
UNCOOPERATIVE
BORED
IMPATIENT
INDEPENDENT
THOUGHTFUL
FOCUSED
FRUSTRATED
DETERMINED
CONFIDENT
EXHILARATED
INTERESTED
ACTIVE
HESITANT
ANXIOUS
HOSTILE
CALM
DISTRACTED
ASSERTIVE
CURIOUS
HARDWORKING
HAPPY

PHASE II. One-on-one "interviews" (the think-aloud protocols) were videotaped as the students solved new versions, both closed and open, of their original test question.

Of course, in adult education, things seldom go according to plan. I began the project with twenty-two students. Over the course of my research project, ten of the original students left the program, and five new students began. Due to The Care Center's policy of rolling admissions and our students' youth and difficult life situations, there is normally a 25-50% turnover each month. However, because of the unusually high turnover (it was, after all, July) and the students' normally erratic attendance, I was only able to gather complete data on twelve of the young women. And since math class only meets once a week, of those twelve students, eight of them ended up doing their think-aloud protocol *before* the written exam.

So what did I learn from this particular process? As I stated earlier, I learned a lot about asking questions. I learned both by reviewing the results from the written test questions and by experiencing the think-aloud protocols.

I had thought that I understood that each person experiences math problems differently. *I* had viewed the GED test question #1 as a measure of the student's knowledge of fraction multiplication.

I thought problem #15 tested whole number division.

#16 I viewed as a multi-step whole number question.

And #20 tested the concept of perimeter.

However, as I tried re-writing the problems other aspects of the questions became clear. How important is context? What role does vocabulary and the assumed language of word problems play? What additional problem solving skills and math concepts are hidden in each problem? In order to rewrite the selected GED problems accurately, I needed to reexamine the problems with more critical eyes.

The first problem *(Figure 1) was* about fraction multiplication. Specifically it included converting a mixed number to an improper fraction, multiplying and then converting the resulting improper fraction back to a mixed number. It also was in the context of sewing materials. I decided to keep the complexity of the question and its context constant throughout the testing, which wasn't difficult. The closed problem for the written test was:

Figure 1

1. To make 5 sets of curtains, when each set requires 2 1/3 yards, how many yards of material are needed?

(1) 7 1/3
(2) 9 2/3
(3) 10 1/3
(4) 10 2/3
(5) 11 2/3

Form BB, © 1987, American Council on Education

Ellen wants to make 3 sundresses, one for each niece. Each dress takes 1 3/8 yards of material. How many yards of material are needed?

and the closed question I wrote for the think-aloud protocol was:

Avery sews 15 stuffed rabbits for a craft fair, each rabbit takes 5/8 of a yard of material, how many yards of material are needed?

To open up the problem, however, proved complicated. For the written one I provided what had been the answer and asked the students to formulate the question:

You have 9 yards of material to make some shirts and pants. Shirts take 1 1/4 yards of material and pants take 2 1/2 yards. How many shirts and pants can you make?

and for the other I tried to write a problem which offered multiple methods for solving:

Tom's new apartment needs curtains. He wants to make 4 pairs of blue curtains for the dining room, 2 pairs of yellow ones for the kitchen and one flowered pair for the bathroom. All the windows are 43 inches wide and 68 inches long so each pair of curtains requires 3 3/4 yards of material. How many yards of material does he need to buy?

What happened was that, in the first case, the students did not notice the restructuring, so they applied the arithmetic with which they'd solved the closed problems to the new set of numbers (ie: 9(1-1/4) + 9(2-1/2) = 33-3/4; it was not clear to me if it was this student's expectations of what math problems should be or if it was some difficulty she had with the language of the problem which caused her error). Then with the open-ended think-aloud protocol question — which could have been solved either by multiplying then adding: 4(3 3/4) + 2 (3 3/4) + 3 3/4 or by adding then multiplying: (4 + 2 + 1)(3 3/4) = 26 1/4 yds — I added extraneous information (the size of the window). I thought it added context and clarity. *Wrong*; it confused both of the students who solved this problem.

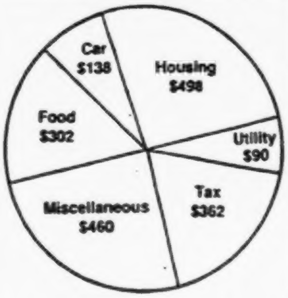


Figure 2

Smith Family Expenses

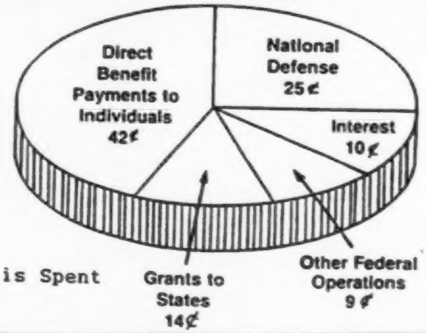
15. Approximately how many times as much money does the Smith family spend on miscellaneous items as it spends on utilities?

(1) 4
(2) 5
(3) 9
(4) 15
(5) 50

Form AA, © 1987, American Council on Education

As I tried to rewrite problem #15 (*Figure 2*), it became clear that it was not simply a whole number division problem, it also required the skills of reading a circle graph. I felt this was an important element to the problem, so I used graphs in the closed versions as well:

About how many times as much money does the government spend on direct payments to individuals as it spends on interest? Refer to the diagram below:



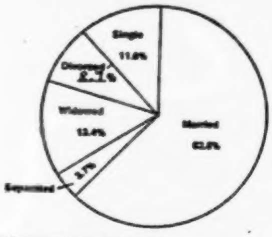
How the Tax Dollar is Spent

How the Tax Dollar is Spent

I used it on the written test and for the think-aloud protocol I asked:

Approximately how many times as many people are married as are divorced?

AMERICAN FAMILY CHARACTERISTICS: ADULT MARITAL STATUS



Source: U.S. Bureau of the Census, 1980.

As I adapted the two pie charts, the first from Form BB of the practice GED test, and the other from *Number Power 5*, I remembered to replace the percents with cents in the first graph, but forgot to eliminate the decimals and percents from the graph for the interview problem. Fortunately, this proved not to be an issue because the think-aloud protocol process allowed me to identify the student who was having trouble with decimals and did not understand the word "approximately" in this context (this misunderstanding was not at all clear from the GED test question, which she got correct; I don't know how which is why I would now also ask a multiple choice question during the think-aloud protocol were I to repeat the project). I was able to separate these difficulties she had and distinguish them from her ability to understand the question as a whole and her agility with the long-division algorithm.

However, I could not see any way to open up the problems, and still include the graphing skill, so I chose to rewrite the problems focusing on comparing two expenditure amounts which the students would determine. The problem for the written test was:

Suppose you have $100.00 to spend on entertainment. How would you spend it? How many times as much money would you spend on your favorite type of entertainment as your least favorite?

And for the think-aloud protocol:

You have been granted the power to decide how the government should spend its money fighting AIDS. How much would you spend on these categories: Education, Medical Research, Medical Care for HIV/AIDS Patients, Other Services for HIV+ People, and Prevention (you may add to the list of categories if you want)? Compare how much money you spent on Research with how much money you spent on Education.

Again, the think-aloud protocol allowed for deeper insight into my students' understanding of the way I wrote the problem. One of the students chose to solve the problem using percents because the total budgeted amount was not known. Still, it appeared that she had a fundamental misconception regarding percents because her budget totaled 150%. However with further clarification available only through think-aloud protocols, it became clear that the student thought that the category "Education" meant GED instruction, so the amount spent on that did not need to be included in her AIDS fighting budget. It was reasonable to spend 150%: 25% on AIDS Research, 50% on Medical Care and 25% on Miscellaneous (totals 100% of the AIDS budget), and for GED Education she would spend the same amount of money as she would for the medical care of HIV/AIDS patients, 50%.

Problem #16 *(Figure 3)* turned out to be a very complicated multi-step problem. It included division as well as multiplication and addition. I think I was able to reproduce the complexity fairly consistently in the closed problems. On the written test:

You are making your mother a bouquet of flowers for Mother's Day. You decide to buy 3 roses, 6 carnations and 24 daisies with greens and baby's breath. They cost as follows:

roses: $2.00 each or 3 for $5.00
carnations: $1.00 each or 3 for $2.00
daisies: 12 for $6.00
greens and baby's breath: $3.00

Figure 3

16. Jim and Alice Moore decided to plant a garden with 3 hosta plants, 6 lavender plants, 60 tulips, and a dogwood. The cost of the plants is as follows:

hosta: $3.00 each or 3 for $8.00

lavender: $1.00 each or 3 for $2.00

tulips: 10 for $4.00

dogwood: $14.00 each

What is the least amount they could spend to purchase the plants they have chosen?

(1) $62.00
(2) $53.00
(3) $50.00
(4) $36.00
(5) $28.00

Form AA, © 1987, American Council on Education

What is the least amount you could pay?

was the problem and for the interview I used:

Evelyn is making a fruit salad for the picnic. She needs 4 pears, 5 oranges, 8 peaches and a mango. The prices are:

pears: 2 for $1.00
oranges: $0.70 each or 5 for $3.00
peaches: $0.30 each or 2 for $0.50
mangos: $2.00 each.

How much did she pay?

The problem also converted easily to open-ended problem solving. Instead of asking the student to find the total cost given a shopping list, the problem became, "What is the shopping list when you can spend a certain amount of money?":

Suppose that you have a clothing allotment for your son of $200.00. The prices at the store are:

shirts: $15.00 each or 3 for $25.00
pants: $35.00 each or 2 for $60.00
underwear: 3 for $5.00
sneakers: $30.00
Adidas or Nikes: $120.00

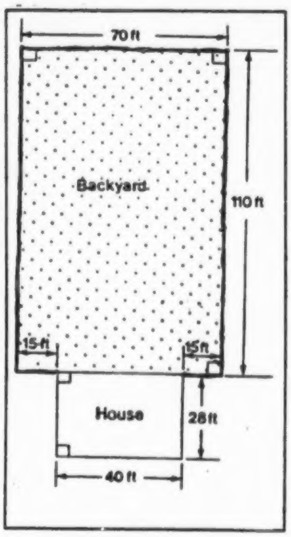
What would you buy and how much would it cost?

was the problem for the written test, while the open-ended problem I wrote for the think-aloud protocol was:

For next year's dance there is a budget of $75.00 for decorations. What would you make and what would you buy? How much would you spend?

colored tissue paper: $3.00 each or 2 for $5.00
streamers: $1.00 each or 3 for $2.00
pipe cleaners: $2.00 for a package of 25
balloons: 50 for $5.00
helium: $12.00 per tank
capia materials (for 50 capias): $15.00
finished capias: $0.50 each

Figure 4



20. Which of the following expressions represents the number of feet of fencing needed to enclose the backyard shown in the diagram above?

(1) 2(110) + 30

(2) 2(110) + 70 + 30

(3) 2(110) + 70

(4) 2(110 + 70 + 30)

(5) Not enough information is given.

Form DD, © 1990, American Council on Education.

Both of these problems allow for multiple answers and ask the students to make value judgements. The only difficulty the students had with how I rewrote these problems, was the "capias". All the students knew that they were small pins given as mementos of special parties. What they didn't understand was why anyone would buy them ready-made (I did not know that the whole point of the pins was the work and love that went into making them. This *I* learned from the think-aloud protocol).

Question 20 *(Figure 4)* refers to the diagram to the left.

What I had initially perceived as a perimeter problem, in fact also involved diagram reading skills and flexibility with the application of formulae and notational understanding. I couldn't see how to include the notation in any of the alternative problem formats because it was imbedded in the multiple choice answers, so I was unable to evaluate that skill. However, I did include diagrams in both closed versions of problem #20:

Dara is going to put a lace border around the frame of her wedding picture (except where the stand is). How much lace should she buy? (Fig. 5)

and for the think-aloud protocol:

Emma is putting weather stripping around her windows and she won't put any where the latches are. How many inches of weather stripping does she need for each window? (Fig. 6)

The open-ended problem for the written test also used a diagram:

Hector has 20 feet of fencing to put around the new garden he's planning for his yard. Where should he put the garden and the fence? (Fig. 7)

Most of the answers I got back on this problem were unclear. Because this question was on the written test, I was unable to determine what the students' answers meant, even the student who answered "I don't understand".

The wording of the think-aloud protocol question also proved to be a stumbling block for the students. Listing all the ways a piece of wallpaper could be used did not clarify the problem, rather it added to the confusion:

> ***You were given a 30 foot roll of a beautiful wallpaper border. You are to use it at home or at school, to decorate around the windows and doors or running along the walls up by the ceiling, at waist level and/or the floor. Explain how you decided to use it and how much you used.***

In addition, I wanted the *student* to create the diagram for this problem. Since that was my intent, I should have written the final sentence to read, "Using a picture, explain..." The reason I didn't was because I was afraid that requesting a diagram would be too directive for the think-aloud protocol process. I didn't want to limit the students' possible approaches to solving the problem. Instead, the students missed the whole point of the problem. The answers I got back were, "I'd put it around the ceiling of my

Figure 7

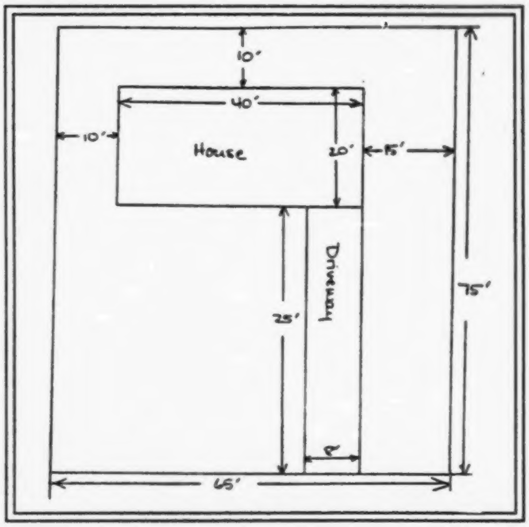


Figure 5

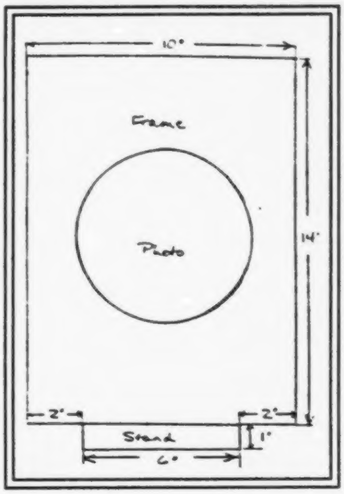
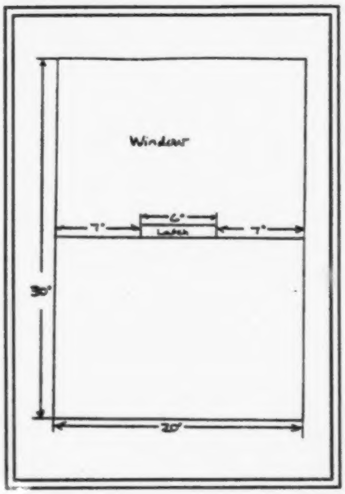


Figure 6



daughter's room," when what I'd expected was answers which explained diagramming and calculating the perimeter of the baby's room. Fortunately because of the think-aloud protocol process, I could ask follow-up questions like, "Do you think there will be enough for her room, or will there be some left over?" These questions prompted some of the students to think about dimensions, but only one ended up making a diagram.

Through experiencing the process of writing test questions and reviewing their resulting solutions, particularly in the context of the think-aloud protocol, I came to appreciate in a deep and personal way how broad the possibilities are for *mis*understanding as well as understanding. As stressed in the *NCTM Standards*, mathematics doesn't exist in a void. It is important, therefore, to bear in mind the student's facility with language, her familiarity with the dominant culture and her assumptions regarding the scope and goals of mathematics and mathematics assessment. I found that, whenever possible, careful verbal questioning should be employed to separate these factors from genuine mathematical conceptual limitations.

I developed effective verbal questioning as I became more practiced with the think-aloud protocol technique of evaluation. Initially, I was reluctant to depart from the problem as it was written to evaluate my students' mathematical knowledge, for fear that I wouldn't be able to make valid comparisons with the other testing methods. However, as I learned to be comfortable using questions to *learn* about my students understandings and not using questions as a method of *instructing* them, I became more assured that leaving the initial question behind was appropriate in many instances. In addition, many of the students were initially confounded by the new process. They didn't know how to think out loud. They felt uncomfortable expressing themselves. And when I would ask a question like, "How do you know it's a division problem?", they assumed that it meant they had made an error and they would immediately begin multiplying instead. It was essential that I began each think-aloud protocol session by explaining not only the purpose of the interview, but also the new process. I reassured the young women by explaining that it was a new experience for me as well as for them; therefore it would seem weird to them at first. I hoped, however, that as we worked through the half-hour of doing math aloud, we would learn how to make it work for us.

I plan to continue to use think-aloud protocols, even though each one was quite time consuming, because it proved to be such a valuable assessment tool. I also hope, with continued exposure, the students will begin to internalize the think-aloud protocol process and be able to ask themselves the probing questions particularly when they are working independently. Some questions which I used frequently were:

To prompt for the student's vocalization of her thinking:

"What are you thinking right now?" "Try to think in slow motion so your mouth can keep up." "I noticed that you were doing something with your fingers. Could you explain it to me?" "What part of the problem are you reading again?" "What was the rule you used in your head?" "Where did that number come from?"

To help the student begin, or get going again:

"What do you think you might have to do?" "What is the problem you're having?" "What part of the problem is giving you trouble?" Repeat the student's question/statement. "Why is that question/statement important?"

To clarify the student's reasoning:

"What makes you think that?" "I don't quite get what you mean." "Here you added, there you multiplied, what's the difference?" "Can you show me how you got that?" "Can you draw a picture?" "You have two answers, which do you like better? Why?" "Can you tie your answer back to the picture?" "Could you repeat what you just said?"

To check the student's understanding:

"Is that the end of the problem?" "What do ***you*** *think it means?" "How did you know that the problem was asking you to add/subtract/multiply/divide?" "How confident are you of your answer?" "How do you know*

when you're right?" "How do you know when you're done?"

All of this information about questions, both written and verbal, was a bonus to the original goal of the project which was to compare what I could discover about my students math understanding from the evaluation tools themselves.

So what did I learn about the various testing methods?

I confirmed my belief that the GED practice test offers little opportunity to develop a sense of my students' mathematical understandings.

I confirmed my belief that the GED practice test offers little opportunity to develop a sense of my students' mathematical understandings. Therefore how they perform on those tests does not provide information I can use to improve instruction. However, I do believe that the GED practice tests are the best indicator of the students' progress toward getting their high school equivalency diploma. Students *can* use the practice test scores as a measure to indicate their progress towards achieving their goal of passing the GED. The only time when one of my student's practice test scores was dramatically higher than her actual GED score proved to be due to her acute test anxiety.

Wouldn't it be nice if there were some other method by which that student and others like her could demonstrate their considerable mathematical abilities?

OPEN-ENDED QUESTIONS HAVE LIMITS, TOO

Unfortunately, from my project I learned that merely adding open-ended questions to a written test does not yield more information, as I had hoped it would, nor does it reduce anxiety. I did not learn any more about my students' mathematical abilities from the open problem than I learned from the closed one on the written exam. However, with both types of questions, I was able to see which students could successfully choose and manipulate arithmetical algorithms, a piece of information I was unable to discern from the GED questions (the student could be a lucky guesser). But that was all.

I believe that the failure of the open-ended problems to yield additional information about my students' mathematical development was in large part because my students had not experienced that type of question before in the context of evaluation. My students also haven't had much practice solving open-ended problems in the classroom. Open problems are most frequently reserved for the weekly "puzzler", an optional exercise. The failure on my part to properly prepare the students for open-ended problems is a clear example of why the alignment standard for evaluation was written: If I expect my students to be able to respond to *any* type of evaluation tool, I need to offer them many safe opportunities to experience that style of questioning. Therefore, I intend to ask my students more open-ended questions because I do believe that open questions can provide the students with rich opportunities to develop deeper insight into various problem solving strategies. By using open questions more frequently in my class, the students can learn from each other's solutions without having to discount their own answer. These types of questions can be used to build the students' self-confidence as well as develop their ability to reason and communicate mathematically. I believe that if open-ended problems were added to the GED test, and evaluated in a similar manner as the GED essay question, it would promote teaching that is aligned more closely with the *NCTM Standards* because if it's on the test, teachers will teach it.

The think-aloud protocol yielded the most information about my students' mathematical understanding. It turned out to be an opportunity to spend thirty to forty-five minutes exploring mathematics with each of my students. It makes sense that I learned the most about what my students understood about math from this luxury of time and lack of restriction. However, one thing which I did find remarkable about the results from the think-aloud protocol was that in this setting the type of question that was posed *did* make a difference. Unlike the results from the written test, when the open-ended problems were not more valuable than the closed ones, in the context of the think-aloud protocol I learned a lot

Unlike the results from the written test, when the open-ended problems were not more valuable than the closed ones, in the context of the think-aloud protocol I learned a lot more about my students' mathematical understandings from the open-ended questions.

more about my students' mathematical understandings from the open-ended questions. The closed problems allowed little room for mathematical exploration for the students who were adept problem solvers. However, by the very nature of open questions, exploration was possible regardless of the level of the student. In fact, the upper level students seemed to enjoy the challenge and depth which the open-ended problems provided. Nonetheless, I think I could have learned more about all of my students if I'd had made manipulatives (like counters, fraction bars, rulers and calculators — all of which we do use in class) available.

From all that I have learned throughout the process of this project, I would say that one of the things I have come to truly understand is that The Care Center's students are being evaluated continually *and meaningfully* on their mathematical understandings. Specifically, during the independent study time (the first two hours of each day), the young women receive frequent informal evaluation. Every day each student gets five to fifteen minutes of one-on-one attention from one of the teachers in the classroom and because of the type of questions we ask, frequently the experience is a mini-think-aloud protocol. Admittedly, the subject which the student is working on at that particular time isn't always mathematics, but we do encourage each student to connect her math understandings to her Social Studies, Science, Literature and Writing work. I have learned that this type of evaluation, whether a formal think-aloud protocol or an informal check-in, is most informative. It is this *process* which provides the most insight into the students' understanding (and misunderstanding) yielding valuable information which I can use to improve mathematics instruction.

---

[1] Two short and user-friendly resources were: *Assessment Alternatives In Mathematics, An Overview of Assessment Techniques that Promote Learning* by EQUALS and *Mathematics Assessment, Myths, Models, Good Questions, and Practical Suggestions* by NCTM. I also used the *Adventures in Assessment* series by SABES, System for Adult Basic Education Support, Massachusetts.

THE RIGHT ANSWER...THERE IS MORE THAN ONE

ADULT DIPLOMA PROGRAM MATH RESEARCH

By Susan Barnard and Kenneth Tamarkin,
Somerville Center for Adult Learning Experience (SCALE)

This project was a collaboration of Kenneth Tamarkin and Susan Barnard. Kenny is a member of the Massachusetts ABE Math Team and Susan is the Program Administrator for the Adult Diploma Program (ADP) at SCALE, in Somerville, MA.

> ***"Decisions concerning the students' [mathematical competency] should be made on the basis of a convergence of information obtained from a variety of sources. These sources should encompass tasks that accept and accommodate the wide range of problem solving strategies represented by a diverse learner population.***
>
> MASSACHUSETTS ADULT BASIC EDUCATION MATH STANDARDS, STANDARD 12: EVALUATION AND ASSESSMENT

We wanted to update SCALE's ADP Math Assessment to reflect the *NCTM Curriculum and Evaluation Standards for School Mathematics* and the *Massachusetts Adult Basic Education Math Standards*, specifically the standard "mathematics as problem solving." The premise of the ADP is to earn a high school diploma by demonstrating proficiencies in life-skill based competencies that have components of reading, writing and math interwoven throughout the curriculum. In order to start the ADP process, students must have an eighth grade reading comprehension level *(Form 4B of Nelson)*, if a native English speaker, and a seventh grade reading level if a non-native speaker; an eighth grade math level; and the ability to write a paragraph with 90% accuracy in structure, grammar and spelling.

Over the years, we have identified the weakest skill area to be critical thinking. We found that we can teach to the ADP math entrance test and students can quickly pass. But given a similar calculation, or problem out of context or with a change in the format, students are not transferring their skills. This project was an opportunity to see how students would respond to a question that could have more than one answer. After all isn't that what life is all about?

In our plan to implement the project, we had targeted students who attended ADP intake/orientation sessions, during which they completed an ADP math pre-test, so we could establish some indication of the person's current mathematical abilities. As we became more involved in the project, however, our target group expanded to include an ABE class, an ADP preparation class, a class at Quinsigamond College, and a few staff members. In total we involved 36 participants in our project: 30 students; 6 staff. The profile of the students varied greatly in reading levels, nationalities, ages, and educational backgrounds. The common factors were that all students were over 18 years of age, all had the goal of achieving a high school credential and all were motivated to attend classes without stipends.

CREATING MATH QUESTIONS

To address our goal of including more problem solving skills in the ADP math curriculum, we developed a set of six open-ended math questions, all of which had a range of correct answers. In order to focus on problem solving rather than computational skills, we wrote all the questions so that they could be correctly answered using whole numbers, though some participants chose to use decimals or fractions in answering some problems. The directions stated that a problem could have more than one answer, but only one answer was required to be written down. Our criteria for success were to have questions that both drew the participants in and revealed their thinking. A follow-up interview sheet was developed to gather individual input from the participants—including their reaction to the test questions, the test format and the test purpose. We hoped to find out whether they considered the new questions fair or unfair, easy or hard, and clear or confusing. We particularly wanted to find out if they enjoyed the new questions more than the traditional test. We also hoped to find out if the participants recognized that the new questions were meant to challenge their thinking skills. Each participant was individually interviewed and the answers recorded on the interview sheet.

Our criteria for success were to have questions that both drew the participants in and revealed their thinking.

The chart on the last page provides an overview of our test results, the score on the ADP math pre-test and the new questions, along with a graph displaying the relationship of pre-test and new questions scores. To understand the relevancy between this test and the six new questions we asked, it is helpful to understand the scoring. On the pre-test, the highest possible score is 28.

If students score between 18-28, a quick 1-3 hour math review usually brings their math abilities to the level where they can pass the math entrance diagnostic; a score of 10-17 usually requires a 6-week class, two hours/week to reach this level; a score between 0-5 usually requires an ABE class from 3 months to a year. On our six math questions, the highest score could be 6, with credit given for partially correct answers. As we began scoring the six questions, we found it necessary for one person to do the scoring in order to achieve consistency. With more than one answer possible and partial credit given, each scorer had a different bias. As a follow-up task to this research, we need to standardize scoring criteria, especially for partial credit.

When asking people to participate, we said we were thinking about revising the ADP math curriculum and needed their help and input. None of the test sessions were timed; students were assured there was no penalty for guessing. The participants' willingness to work with us and give us honest and complete feedback was outstanding. Word of the project spread and we had teachers asking if they could try the questions with their students. (Those results are not included in our findings.)

We decided to write up the results of our findings separately and then meet to discuss our findings. To our surprise, our write-ups were strikingly similar.

Question #1: You have a $100 to share among three people. You don't have to divide the money equally, but no person can have less than $25. How much does each person have?

We intentionally picked numbers that couldn't be divided evenly and stated the three people did not have to get the same amount. Yet, we found that most participants wanted to be "fair" by dividing the money equally in three ways, giving each person $33, or even $33.33, often losing sight that the amounts, once divided, should add up to $100. One participant, while being interviewed, commented, "Of course we divide everything evenly. We're parents."

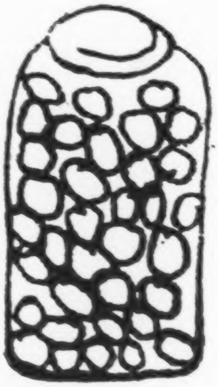
Question #2: You go to the grocery store with $25 in your pocket to buy milk, orange juice, bread, and shampoo. What other items would you buy so that you would go home with no less than $10 in your pocket? How much money do you have left?

Grocery items:

bread	*$1*	*cookies*	*$2*	*spaghetti*	*$1*
tooth paste	*$2*	*cat food*	*$1*	*cucumbers*	*$1*
cheese	*$4*	*milk*	*$2*	*pancake mix*	*$1*
orange juice	*$2*	*fish*	*$4*	*ice cream*	*$3*
lettuce	*$1*	*lemons*	*$1*	*shampoo*	*$4*

People seemed to particularly enjoy this question, probably because people love to shop. If they bought the required items they would have $16 left, which was a correct answer. Yet, people seemed to focus on the $10, opting to buy more, going home with exactly that amount. Other people did not organize or label their work and became confused, sometimes thinking $10 more needed to be spent and losing sight that they needed to *go home* with $10. Overall, this question had the highest rate of correct answers.

Question #3: There are 10 pieces of candy in one jar. Estimate the number of pieces of candy in the other jar.



The other jar contained 105 pieces of candy wrapped in cellophane. The idea was to see if people's answers would be in the ballpark. We decided on a range of 30 pieces in either direction would be an accepted answer (because Kenny guessed 130 and didn't want to be wrong.) A surprising number of people got this answer wrong; quite a few answered 70. The problem was the paper added extra volume and the candy could be tightly or loosely packed, making it difficult to estimate accurately. Despite the problem with the cellophane, we were still able to get some valuable information; participants who guessed 50 or below also had severe problems with computation and seemed to lack basic number sense. This was the hardest question to control because participants kept eating the candy. We would alter this question if we make it a permanent part of ADP assessment. We would use M&Ms (preferably peanut, because they are bigger and taste better), beans or some other unwrapped, regularly shaped, small object as the standard so that the volume estimation could be more consistent.

Question #4: Draw a rectangle whose perimeter is 8 inches.

In this question, participants had to know what a rectangle and perimeter are. The standard approach to this question is: What is the perimeter of a rectangle whose sides are 3, 1, 3, 1? The question also emerged, "Is a square a rectangle? Yes, it is. There were two whole number correct answers: A square with 2 inch sides and a 3 inch by 1 inch rectangle. Most people drew the 3 by 1 rectangle. We also accepted as correct a 2-1/2 by 1-1/2 inch rectangle, since we did not require that the answer had to be in whole numbers. One-third credit was deducted if the lengths of the sides of the rectangle were not labeled, but if we implemented this question, we would directly ask for the sides to be labeled, since this requirement was not entirely clear. A number of people did not even attempt this question, considering it too hard. Others drew 8 by 1 inch or 2 by 4 inch rectangles, mistaking area for perimeter.

Question #5: A room is 15' by 12'. You need to place a wood burning stove that is 3' x 2' in the room. The stove must be at least 1' from any wall. Draw a floor plan that shows where the stove can be located.

This question encouraged more creativity. Although there were many potential correct answers to this question, many participants did not even attempt it, again stating that the question was too hard. Some people did draw good floor plans, but did not label the dimensions, missing the importance of clearly communicating their conclusions. During the interview, participants said they wondered if the plan should be drawn to scale or if they should label things. We would alter the question to make clear that the dimensions and distances need to be written on the diagram.

Question #6: 250 - 74 + 120 = 296 . Using these numbers, write a word problem.

This question also required a bit more creativity and was fun to read from an evaluator's perspective. Some of the answers were quite entertaining and creative, as well as being correct, but many of the people had difficulty writing an accurate word problem. Participants had the most difficulty with phrasing a question that would lead someone to writing an answer. Instead, many included the answer in their word problem. For example, one word problem said, "I have 250 $ in the Bank. I took 74 from it to pay my bill, and the next week I deposit 120 now my balance from the bank is 296 $." We feel that clarity in the directions is needed. Something like: *The answer should not be stated in your word problem.*

On our math survey we asked participants to rate the problems from 1 to 5 in the following categories: Fair to Unfair, Easy to Hard and Clear to Confusing. Most students thought that the questions were Fair and Clear, responding to these questions with a 1 or a 2. The Easy to Hard question mostly received 3's.

During the interviews, people who did best on the traditional test and very well on the new questions tended to be particularly enthusiastic. Some comments were "It makes you think", "It was fun." The few negative reactions came mostly from people who had done poorly on both the traditional test and the experimental questions. They had comments such as "it was confusing," and a number of these poor math achievers were unsure exactly why we were trying to do this project. However, overall, we were pleased to find out that almost everybody was very cooperative and actually enjoyed being asked for their input before changes were implemented.

ANALYZING THE RESULTS

In analyzing our statistics, we found that of the 14 participants who got 1/2 or less of the ADP pretest correct (14 or below), only 4 got 1/2 (3 or more) correct on the new questions; only 1 got 4 out of 6 correct. Of those 16 participants who got more than 1/2 correct on the ADP pretest (15 to 28), only 2 scored below 1/2 correct (less than 3) on the new questions.

The results raise some interesting questions. Is there a significant correlation between computational skills and reasoning skills? Can someone who was unsuccessful learning math in a computation-oriented program learn more effectively in a problem-solving oriented program?

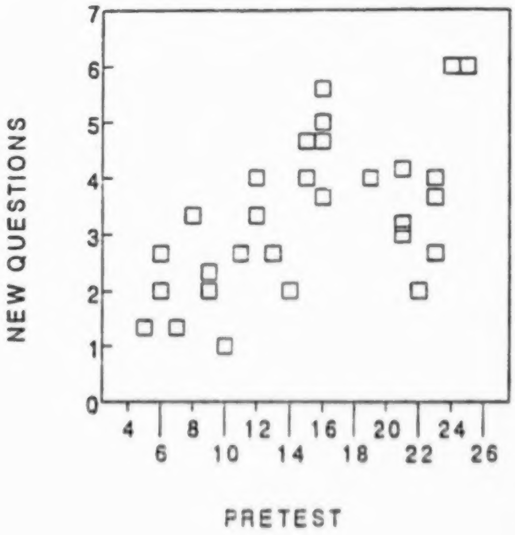
The open-ended questions were definitely more interesting to score. The variety of answers and approaches gave great insight into how the student was thinking. The wrong answers were particularly helpful in showing how an individual approached a problem, and also when the working of the problem was open to misinterpretation. All our questions met our criteria; we just need to refine and revise or directions.

Our conclusion is that we should proceed with altering the ADP tests and curriculum to include open-ended questions and problem-solving skills. We believe that this will create a richer, more relevant experience that will translate well into improved functioning and critical thinking in everyday life. We also thought that it was important to give people options in the testing situation, since that more closely reflects real life than exclusively having questions with only one correct answer. The correlation that we observed between achievement on the computation oriented exam and the problem-solving oriented questions also suggests to us that there is a role for effective computational instruction and that progress in problem solving skills can indeed be meaningfully measured and evaluated.

ADP MATH ANALYSIS

NAME	PRETEST	NEW QUESTIONS
JD	5	1.3
AG	6	2.0
TK	6	2.7
MX	7	1.3
DJ	8	3.3
CG	9	2.0
MK	9	2.3
TR	10	1.0
MB	11	2.7
KX	12	3.3
MP	12	3.3
SE	12	4.0
AV	13	2.7
DG	14	2.0
AS	15	4.0
CJ	15	4.7
GS	16	3.7
IG	16	4.7
ML	16	5.0
PSJ	16	5.6
MA	19	4.0
B	21	3.0
RC	21	3.2
MB	21	4.2
DG	22	2.0
JDF	23	2.7
CD	23	3.7
SP	23	4.0
NP	24	6.0
MV	25	6.0

ADP MATH ASSESSMENT



ABOUT THE MASSACHUSETTS ABE MATH TEAM

Bill Arcand is the regional coordinator for the System for Adult Basic Education Support (SABES) and provides staff and program development to all the ABE learning centers in Western Massachusetts.

Leslie Arriola recently completed a doctorate in education. Her dissertation research focused on adults and college developmental mathematics. She has a lifelong commitment to mathematics reform and has been a long-time supporter of the *Standards*' philosophy.

Susan Barnard has been involved in adult education for twenty years and is currently the administrator of the Adult Diploma Program at SCALE (Somerville Center for Adult Learning Experiences).

Shelley Bourgeois combines administrative and teaching roles in a Boston community-based organization. She recognizes quality thinking and quality foods as soon as she sees them.

Catherine (Cathy) Coleman makes a full-time living out of part-time jobs. She has coordinated the student-writing anthology, "Do the Write Thing," in the Central region of Massachusetts for three years.

Donna Curry is a consultant to workplace education programs. She thrives on teaching statistics and probability. Her publications include "Quality Quest" and "Teamwork: the Quality Message."

Karen DeCoster has brought her sense of humor and creativity to many Massachusetts adult education projects. In addition to the Math Team research, she's has participated in MCET teleconferences, and has authored *Curriculum for Government and Law*, currently available to Massachusetts teachers through their SABES coordinator.

Tricia Donovan is dedicated to offering an integrated, social curriculum, though that may mean jumping off the deep end into uncharted waters. She has brought her journalism qualifications to the Math Team as the editor of the statewide newsletter *The Problem Solver*.

Margaret (Peg) Fallon has taught for 15 years and plans to continue. Her enthusiasm for new projects and ideas are a valuable contribution at every Math Team event.

Barbara Blake Goodridge has taught in the Lowell Public Schools for 23 years. She is an active talker and lisetener, fascinated by verbal interaction. It's no accident she chose the Communications standard as a focus of her research.

Linda Huntington. While several of the Math Team members are noted for their creativity, Linda took it one step further by bringing the creative arts together with math in her ABE classroom. She and her students made quilts with math woven in.

Esther Leonelli has been "fighting the good fight" from the beginning. She was one of the original ABE teachers who asked NCTM to extend its agenda to adults in 1990. She plays fiddle and surfs the Internet where she can be reached at edl@world.std.com.

Thomson Macdonald is the full-time study center coordinator at the Haitan Multi-Service Center. He has made great strides in bringing computers into the classroom. As funds have become available through the PALMS project, Tom has brought computer literacy and the Standards together at the center.

Martha Merson lives to swim and eat chocolate. She lives in Jamaica Plain. Her work involves finding ways for talented teachers to learn from each other.

Marilyn Moses lives and teaches in Brockton where she grew up. She avoids writing and animals and loves to sew. Her blueberry preserves are to die for.

Adrienne Morris manages the SABES Office in Bill Arcand's absence. As the Project Assistant, she coordinated meetings/conferences, served as a clearinghouse of all data generated and was responsible for disseminating it among the subgroups. Most importantly, she generated voluminous paperwork to get members paid!

Debra Richard was a traditional teacher but has changed her techniques dramatically. The warmth and understanding she conveys to her students is reciprocated. Debra is a real actress having done three MCET teleconferences and is very comfortable in front of the camera.

Mary Jane Schmitt has been in adult education for 23 years. She is known for her video "Changing the Rules," available through New Readers Press, and for getting up at 4 a.m.

Ruth Schwendeman is known for her work in critical thinking. She brings precision and faith in the public schools and their teachers to all her work. In her spare time, Ruth takes care of her country home and country garden.

Sara (Sally) Spencer lives in Western Massachusetts where she brings a true constructivist approach to teaching teen parents at the CARE Center. She brought the Math Team into contact with the SummerMath for Teachers program based at Mount Holyoke College.

Judith (Judi) Sulzbach is a former first-grade teacher who got roped into subbing for an ABE class six years ago. She's been hooked on adult ed ever since. She splits her time betweemn workplace and community-based programs.

Kenneth Tamarkin is currently creating GED software for Southwestern Publications. Ken is also a published author of Number Power 6 and Pre-GED Social Studies [Contemporary Books].

Leonora Thomas is dedicated to adult education. She is extremely innovative and was recognized by the New Bedford News with a full-page photo essay on her part in the PALMS project.

AFTERWORD

As far as our math team is concerned, this is just the beginning. There is much that needs to be done in order for our vision of quality math education for all adult learners to become a reality. There are few instructional materials on the market that encourage communication, reasoning, and problem solving in realistic settings. Good **materials** that help implement the standards need to be developed. Then, **staff development** that encourages teachers to take off their expert hats and begin to learn math with other teachers and students is needed. We've begun by developing some "hands-on" workshops for our peers.

We're motivated to create **assessment** which does a better job at supporting learning. Finally, we want to continue as **researchers** in our classrooms.

We encourage all ABE/GED/ESL math teachers to join NCTM and attend their conferences, and to form your own local support teams. Change is difficult. It takes time. But, change is easier together than alone.